Stochastic Stability of Public Debt: The Case of Austria

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Abstract

This paper extends a method developed by Hall (2014) to characterize the long-run distribution of public debt and applies it to Austria. We use Bayesian estimation techniques to incorporate data from other countries which makes the model applicable to cases in which available time series is short. We find that the long-run trend of Austrian fiscal policy is in line with the 60% threshold of the Maastricht treaty. Our results suggest that the strong increase in the debt-to-GDP ratio in the aftermath of the financial crisis represents a tail event which, given the long-run trend, does not provide a sufficient cause for rational investors to question the sustainability of Austrian public finances.

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1. Introduction

In recent years the sustainability of public debt has become an issue of great concern in public discourse. The debt crisis in the Eurozone alerted both policy-makers and economists to the fact that government defaults are not exclusively a problem of developing countries. With few exceptions all major Western economies have seen their public debts rise faster than their national incomes. If current demographic trends are to persist, a rise in age-related government expenditures will impose more and more pressure on public budgets in the future. Uncertainty about the future growth outlook adds to concerns about the ability of governments to service their debts. In addition, the Eurozone debt crisis has demonstrated that governments who do not control the currency in which their liabilities are denominated are vulnerable to sudden swings in investor confidence.

Projections of future public debt face several uncertainties. First, there is policy uncertainty regarding the future development of the tax system and government spending. Second, even if one assumes no changes in tax and spending policies, there is economic uncertainty which must be taken into account. The growth rate of GDP, demographic changes as well as the interest rate at which the government can borrow iare part of the economic environment which directly or indirectly affects a country's public finances. Since this economic environment is subject to random shocks, assessing the sustainability of a country's debt requires an estimation of the (joint) probability distribution of the country's economic fundamentals and its public debt level.

This paper uses a method developed by Hall (2014) to characterize the long-run distribution (Hall refers to it as the "ergodic distribution") of the public debt and applies it to the case of Austria. The ergodic distribution is derived from a Markov-chain model of the debt-to-GDP ratio and several key macroeconomic variables. The primary advantage of this method is that it generates not only a point prediction of the debt level but gives probabilities of tail events such as an abrupt increase in the debt-to-GDP ratio above a given threshold. It provides an empirical test of government debt sustainability insofar as the non-existence of a stable long-run distribution (a stochastic steady-state) points to a violation of the government's intertemporal budget constraint.

We study the case of Austria for two reasons. First, Austria is in many respects a "typical" Eurozone country - both in terms of GDP growth and debt-to-GDP ratio - and thus can be seen as representative for the Eurozone. Second, Austria has had a fairly long tradition of "activist" fiscal policy which has raised concerns about long-run debt sustainability (Haber and Neck (2008)). These concerns have been heightened during the recent financial crisis when the Austrian government stepped in to support the national banking sector leading the public debt to exceed 80% of GDP. This paper helps answering the question whether the recent spike in Austria's debt level implies a persistent threat to government solvency or should be seen as a transitory phenomenon.

In addition, this paper makes a methodological contribution by developing a novel procedure to estimate the transition matrix of the Markov-chain model. The maximum likelihood approach taken in Hall (2014)'s original paper runs into problems when applied to short time series which may lead to distortions of the derived ergodic distribution. We propose a Bayesian approach which takes into account prior information on transition probabilities from other countries and thus overcomes the problem of short time series.

The rest of the paper is in three sections. The next sections describes and discusses the setup the of model. Section 3 explains our estimation procedure. In Section 4 we apply our model to the case of Austria and present the debt projections. The conclusion summarizes the main findings of the paper.

2. Related Literature

Assessing long-run fiscal sustainability in the sense of whether or not the government satisfies its intertemporal budget constraint has given rise to a vast amount of papers dealing with this particularly important question of public finance theory. Early contributions (e.g. Trehan and Walsh (1991)) typically test if the historical record of fiscal policy supports the hypothesis that the initial debt does not exceed the expected value of future primary surpluses discounted by the interest rate on sovereign debt. This requires that discounted value of expected future debt converges to zero which is known as the transversality condition. The validity of testing fiscal sustainability from this concept depends on the choice of the discount factor.

Bohn (1995) and Bohn (2008) discuss extensively the problems arising from the approach to use the historical average of the return on sovereign bonds for discounting expected future debt. He shows that in a stochastic general equilibrium setting the marginal rate of substitution, which might be subject to significant variation over time, is the correct discount factor. He further demonstrates that using historical average of the real interest rate for discounting might under reasonable conditions lead to misleading conclusions. More generally, using the historical average sustainability tests is valid only if at least one of the three assumptions holds: i) no uncertainty, ii) risk-neutrality of private agents or iii) no correlation between future government surpluses and marginal rates of substitution. Mendoza and Ostry (2008) argue in line with Bohn (2008) that i) and ii) are unrealistic and that iii) is sharply at odds with empirical findings. Consequently, Bohn (1998) and Bohn (2008) propose a sufficient criterion to assess whether the government satisfies its intertemporal budget constraint which is independent of the underlying asset pricing kernel.

The "Bohn-Test" basically consists of estimating the feedback parameter ρ in the equation:

$$pb_t = \rho \; deb_{t-1} + \mu_t + \varepsilon_t \tag{1}$$

where pb gives primary balance-to-gdp- and deb debt-to-GDP ratio. Additionally μ is used to correct for temporary fluctuations in output and governmental expenditures arising e.g. from extraordinary expenses on military in times of war. Proposition 1 in Bohn (2008) states that a positive reaction parameter ρ reflecting increases in the primary balance in reaction to a rise in the debt-to-gdp ratio is sufficient to guarantee that the government does not run a Ponzi-scheme. Note that this condition does not rely on assumptions about maturity and/or denomination of debt. Governmental securities can take the form of any state-contingent bond. Additionally, the test does not require any knowledge about the tax- and expenditure policies of the government since it solely determines whether their outcome is consistent with the intertemoral budget constraint (see Mendoza and Ostry (2008)). The test is only based on the assumptions of complete asset markets and the existence of a well-defined pricing kernel independent of its particular nature.

Several contributions apply the "Bohn-Test" to the Austrian economy (see Getzner et al. (2001), Neck and Getzner (2001) and Haber and Neck (2006)). All of them find evidence in favour of sustainability. The attempt in Haber and Neck (2006) covers the period between 1960 and 2003. The study finds a structural break in the fiscal reaction parameter ρ between 1974 and 1975 when the first oil price shock hit the Austrian economy. This result is also supported by Mauro et al. (2013) in a recent IMF publication. Haber and Neck (2006) find a strong positive and significant reaction of the primary deficit to increases in public debt levels between 1960 and 1974 and a still positive but considerably smaller reaction between 1975 and 2003.
While several contributions use Bohn's method of estimating fiscal reaction functions to check whether fiscal policy has acted "responsible" in the past, little effort has been made to study the implied future path of the debt-to-gdp ratio. In a recent publication, Hall (2014) provides such an analysis of the trajectory of US public debt and quantifies the implied mean debt-to-gdp ratio together with the tail probabilities that a sequence of crisis years might drive the debt-to-gdp ratio to abnormally high levels from which there is no return to lower ones without changes in fiscal policy. Our approach is largely based on his attempt. The subsequent section provides an overview of the method.

3. The Model

Let the primary deficit be given by D_t and the number of outstanding bonds at the end of the period by B_t. The primary deficit consists of an exogenous component X_t reflecting budgetary shocks such as above-average unemployment compensations or banking bailouts in crisis years and an endogenous component measuring the response of the primary deficit to the debt obligations outstanding at the beginning of the period. Formally:

$$D_t = X_t - \alpha B_{t-1} \tag{2}$$

The parameter α therefore corresponds to the fiscal reaction parameter ρ of the Bohn test. To avoid confusion, we nonetheless use different variable names as the two approaches differ in several important aspects as will become clear soon. Throughout the analysis we mostly express the primary deficit and the outstanding debt as ratios to GDP and always denote the resulting variables by the corresponding lower case letters d_t, x_t and b_t. In terms of GDP ratios, equation (2) thus becomes:

$$d_t = x_t - \alpha b_{t-1} \tag{3}$$

Let the evolution of the economy be characterized by a stochastic process with state space $S = \{s_1 ... s_k\}$ containing a finite number k of fundamental states. Assume further that the conditional probability distribution is solely affected by the current state implying that the underlying process satisfies the Markov property. Let ω give the transition matrix between fundamental states $s_i \in S$ for $i = 1, ...k$ and let x_t, g_t and q_t be the state-dependent realizations of the exogenous component of the primary deficit (x), the GDP growth (g) and the price of sovereign bonds (q). Those bonds are assumed to pay a coupon with start value κ that declines by a constant fraction δ each year. It is meaningful to depart from standard one-period debt obligations as the chosen approach smooths the interest burden resulting from the roll-over of public debt across states of nature. This feasibly reduces the effects of changes in interest rates on the debt-to-GDP ratio in line with empirical evidence that the average duration of outstanding bonds is commonly larger than one year. The value of κ is chosen such that the price of a newly issued bond equals 1. From this specification, b_t closely approximates the debt-to-GDP ratio. The current price of a bond equals the present discounted value of expected future payoffs. From Bohn's critique, one needs appropriate stochastic discount factors to determine the current value of a bond. As the implied future path depends solely on past outcomes and as the analysis covers only one sort of bond, one can directly measure the state dependent discount factor from past average returns on sovereign obligations in state s.

The number of outstanding bonds in period t equals the sum of the number of newly issued bonds in this period and the number of bonds inherited from the period before (δB_{t-1}). New bonds need to cover the ongoing primary deficit (eq. 2) and the coupon payment on past obligations κB_{t-1}. Mathematically:

$$B_t = \frac{D_t + \kappa B_{t-1}}{q_t} + \delta B_{t-1} \tag{4}$$

which, after inserting (2) is equivalent to:

$$b_t Y_t = \frac{(X_t - \alpha B_{t-1})\, Y_t + \kappa b_{t-1} Y_{t-1}}{q_t} + \delta b_{t-1} Y_{t-1} \tag{5}$$

Dividing (4) by Y_t, substituting $g_t = Y_t/Y_{t-1}$ and factoring out b_{t-1} on the right hand side yields:

$$b_t = \frac{x_t}{q_t} - \left(\frac{\alpha}{q_t} - \frac{\kappa}{q_t g_t} + \frac{\delta}{g_t}\right) b_{t-1} \tag{6}$$

The debt-to-GDP ratio depends on x,q and g and is therefore itself a random variable. The actual state of the economy is then given by the fundamental state s and the prevailing level of public debt b. Denote the actual state by $a_t = [s_t, b_t]$. The conditional joint cumulative distribution function T of a_t given a_{t-1} then reads:

$$T(a_t \mid a_{t-1}) = \Omega_{s_t,s_{t-1}} \mathbb{I}\left(b_t - \frac{x_t}{q_t} - \left(\frac{\alpha}{q_t} - \frac{\kappa}{q_t g_t} + \frac{\delta}{g_t}\right) b_{t-1}\right) \tag{7}$$

where $\Omega_{s_t,s_{t-1}}$ is the conditional cumulative distribution function of s_t given s_{t-1}. The indicator function $\mathbb{I}$ gives a zero if the level of b_t implied by the originating value b_{t-1} and destination state s_t exceeds the level of debt which underlies the destination state a_t and 1 elsewhere. Thus:

$$\mathbb{I}\left(b_t - \frac{x_t}{q_t} - \left(\frac{\alpha}{q_t} - \frac{\kappa}{q_t g_t} + \frac{\delta}{g_t}\right) b_{t-1}\right) = \begin{cases} 0 & \text{if } \left(b_t - \frac{x_t}{q_t} - \left(\frac{\alpha}{q_t} - \frac{\kappa}{q_t g_t} + \frac{\delta}{g_t}\right) b_{t-1}\right) < 0 \\ 1 & otherwise \end{cases}$$

Define the cumulative distribution function of the actual states by $Q(a_t)$. It is formally given by:

$$Q(a_t) = \int_{s=1}^{k} \int_{B=-\infty}^{\infty} T(a_t \mid a_{t-1}) dQ(a_t) \; \forall a_t \tag{8}$$

The marginal cumulative distribution function of b_t, say Υ, resulting from (8) then gives the path of the debt-to-GDP ratio implied by the underlying Markov-process together with the "leaning-against-debt" parameter α. Fiscal policies consistent with a stationary distribution do not lead to an explosive path of the debt-to-GDP ratio.

How does α influence the outcome? From the basic theory of Markovian processes, a Markov chain exhibits a unique stationary distribution if and only if it is irreducible and aperiodic. Roughly speaking, this requires that it is possible to reach any state of the $[s_t, B_t]$ matrix independent of the current status and that every state can return to itself without a fixed number of iterations between the two visits. From equation (3), the higher α is, the lower is the primary deficit in the subsequent period. This considerably facilitates the possibility to return to low levels of public debt as the absolute size of the reaction of the primary deficit rises with the accumulation of debt and thus increases the likelihood that the underlying Markov chain is irreducible. [3]

4. Estimation Procedure

The first step in applying our model is to estimate the state means from the observed fundamental variables as well as the transition probabilities among states.

[3] Note that in the Bohn-Test, a positive and significant reaction parameter ρ is sufficient to guarantee a non-explosive path of the debt-to-GDP ratio. In terms of our analysis, α is calculated differently as will become clear in the next section implying that the two methods cannot be compared entirely.

We observe a vector sequence $\{y_t\}_{t=0}^T$ of fundamental variables (inflation, real interest rate, unemployment, real GDP growth). Following Hall (2014) we employ the k-means clustering algorithm to find k clusters in the data which we identify as k fundamental states of nature. Thus we obtain a sequence of integers $\{s_t\}_{t=0}^T$, where $s_t \in \mathbb{N} \cap [1, k]$ is the observed state of nature in period t.

We model the sequence of fundamental states as a discrete, time-homogeneous Markov chain. The conditional probability of s_t given $s_0, ..., s_{t-1}$ obeys

$$\mathbb{P}[s_t|s_0, ..., s_{t-1}] = \mathbb{P}[s_t|s_{t-1}], \tag{9}$$

which is known as the Markov property. Time-homogeneity means that the transition probabilities

$$\mathbb{P}[s_t = j|s_{t-1} = i] = \omega_{ij}, \tag{10}$$

are independent of t. We let $\Omega = [\omega_{ij}]$ denote the $k \times k$ matrix of transition probabilities. Our goal is to estimate Ω from the sequence of observed states.

The conditional probability of $\{s_t\}_{t=0}^T$ given Ω is

$$\mathbb{P}[s_0, ..., s_T|\Omega] = \mathbb{P}[s_0] \prod_{t=1}^{T} \mathbb{P}[s_t|s_{t-1}, \Omega], \tag{11}$$

which is equal to the likelihood of the transition matrix. The likelihood summarizes the information about the unknown transition probabilities contained in the sequence of observed state transitions. As a result of time-homogeneity, $\mathbb{P}[s_0, ..., s_T|\Omega]$ is a multinomial probability mass function with $k \times k$ parameters $\omega_{11}, \omega_{12}, ..., \omega_{ij}, ..., \omega_{kk}$:[4]

$$\mathbb{P}[s_0, ..., s_T|\Omega] \propto \prod_{i=1}^{k} \prod_{j=1}^{k} \omega_{ij}^{N_{ij}}, \tag{12}$$

[4]We will ignore irrelevant normalizing constants throughout.

where N_{ij} is the number of observed transitions from state i into state j:

$$N_{ij} = \sum_{t=1}^{T} \mathbb{I}(s_t = j, s_{t-1} = i), \tag{13}$$

with $\mathbb{I}$ being the indicator function. Maximizing (12) with respect to ω_{ij} yields the maximum likelihood estimator for the transition probabilities:

$$\omega_{ij}^{MLE} = \frac{N_{ij}}{\sum_{i=1}^{k} N_{ij}}. \tag{14}$$

This is the estimator used in Hall (2014). Applying the maximum likelihood estimator in practice can be problematic if the sample of observed states is small. For instance let $T = 50$ and $k = 5$, then $N_{ij} \approx 2$ on average, so $\hat{p}_{ij}^{MLE}$ will assign zero to many transition probabilities. This tendency can be alleviated by explicitly taking into account prior information about the transition probabilities using a Bayesian approach. Prior information may come from theoretical considerations or from previous empirical work. In Bayesian estimation, the likelihood is combined with a prior distribution to yield a posterior distribution for the unknown parameters of the model.

We use the Dirichlet distribution as the prior, because it is conjugate to the multinomial distribution. Each row i of Ω is Dirichlet distributed with parameters $\mu_{ij} > 0$, $j = 1, ..., k$ and independent from all other rows. Hence, the prior probability of the transition matrix is:

$$\mathbb{P}[\Omega] \propto \prod_{i=1}^{k} \prod_{j=1}^{k} \omega_{ij}^{\mu_{ij}-1}, \tag{15}$$

where $\mu_{ij} > 0$ indicates the strength of the prior information about the transition probability ω_{ij}. The posterior distribution is now obtained via Bayes' Rule:

$$\mathbb{P}[\Omega | s_0, ..., s_T] \propto \mathbb{P}[s_0, ..., s_T | \Omega] \mathbb{P}[\Omega] = \prod_{i=1}^{k} \prod_{j=1}^{k} \omega_{ij}^{N_{ij}+\mu_{ij}-1}. \tag{16}$$

The posterior is the probability distribution of the transition matrix, conditional on the observed state transitions $s_0, ..., s_T$. It gives a complete description of the uncertainty about transition probabilities taking into account the information contained in the observed state transitions as well as the information contained in the prior.

Note that the posterior is a Dirichlet distribution with parameters $N_{ij} + \mu_{ij}$. It is therefore straightforward to compute the posterior mean of ω_{ij}:

$$\omega_{ij}^{PME} = \frac{N_{ij} + \mu_{ij}}{\sum_{i=1}^{k}(N_{ij} + \mu_{ij})}. \tag{17}$$

Clearly, the posterior mean estimate is a weighted sum of the prior mean and the maximum likelihood estimate, where the weight reflects the relative strength of the prior:

$$\omega_{ij}^{PME} = \frac{\sum_{i=1}^{k} N_{ij}}{\sum_{i=1}^{k}(N_{ij} + \mu_{ij})}\omega_{ij}^{MLE} + \frac{\sum_{i=1}^{k} \mu_{ij}}{\sum_{i=1}^{k}(N_{ij} + \mu_{ij})}\omega_{ij}^{PRM}, \tag{18}$$

with $\omega_{ij}^{PRM} = \frac{\mu_{ij}}{\sum_{i=1}^{k} \mu_{ij}}$. In the limit, if $\sum_{i=1}^{k} N_{ij}$ becomes infinitely large, the posterior mean equals the maximum likelihood estimator. Roughly speaking, as the sample size grows larger, the posterior mean approaches the "conventional" maximum likelihood estimator everything else remaining equal.

5. Application: Fiscal Sustainability in Austria

We apply the k-means algorithm described above to identify five clusters from Austrian data of inflation (π), real interest rate (r) unemployment (u) and GDP growth (g). [5] Table 1 lists the resulting fundamental states of the economy together with the implied values of the considered variables where the primary deficit (d) associated with each state is assigned after the clustering.

Note that the method is fairly well able to detect the recessions of 1978 and 2009 as well as the oil price shocks in 1975 and 1980 together with the subsequent periods characterized by high inflation and modest growth[6]. In contrast to Hall (2014), unemployment plays

[5] See table (2) in Appendix for details about the employed data.

[6] See table (3) in Appendix for a detailed classification of the years from 1975-2013 with respect to the fundamental state

State	g	π	u	r	d	*Example years*
1	-0.5	2.2	3.8	3.5	-0.47	1978, 2009
2	1.8	6.5	2.4	2.8	0.26	1975, 1980
3	2.2	2.6	4.4	0.7	-0.08	2011, 2012
4	2.9	2.5	3.6	5.0	0.1	1979, 1995
5	3.0	1.5	4.4	3.2	-1.22	1997, 2006

Table 1: Fundamental States (all values in percent of GDP)

only a minor role in the identification process which can be traced back to the strong tendency of Austria's fiscal policy to "lean against unemployment" (see Haber and Neck (2006) for a discussion).

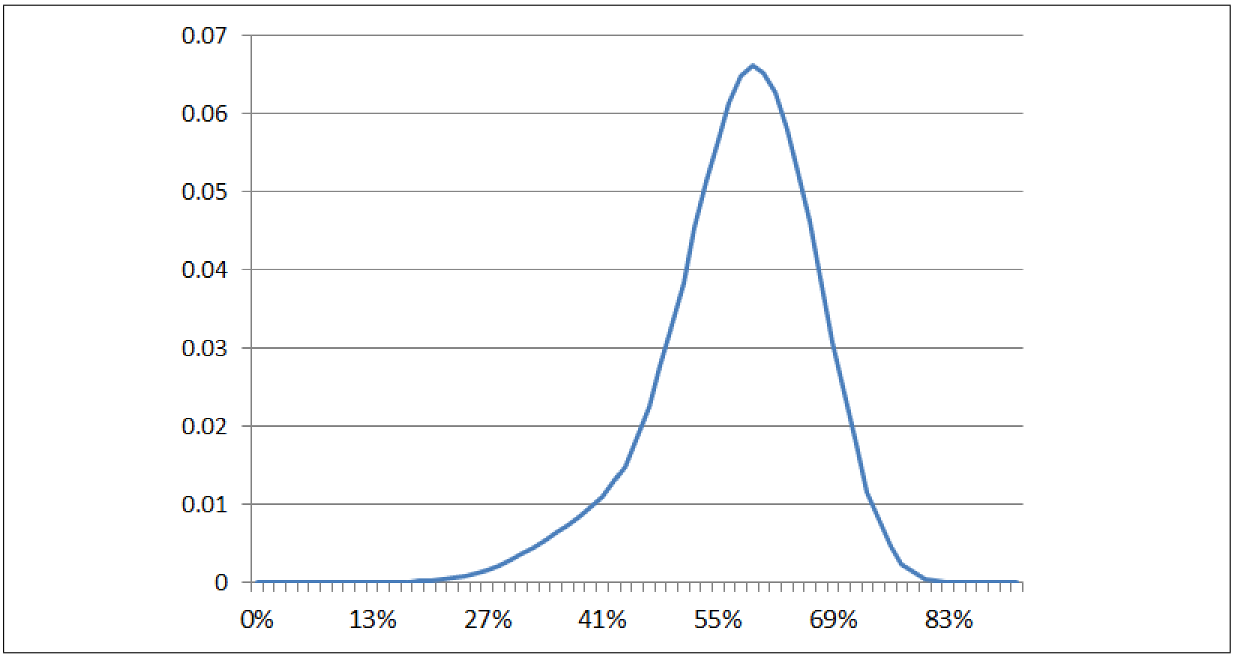

Figure 1: Distribution Function Debt-GDP ratio

A brief look at the evolution of the Austrian debt-to-GDP position over the considered period (see figure (2)) might trigger the fear of ever rising debt as the ratio increased from slightly beyond 25% in 1975 to roughly 80% in 2014. In contrast, the decade preceding the economic downturn after 2007 was characterized by a shrinking public debt level such that the value in 2007 was close to the ones of the late 1980s. Can the decline in the debt-to-GDP ratio be traced back to an abnormally long sequence of prosperous years such that the rise after 2007 marks a return back to a trend increase or does, contrary to this view, the sharp increase after 2007 represent a surprising and temporary event given the historical record of Austrian fiscal policy?

Figure (2) shows the public debt projection starting from state 5 with the actual debt level of 60% in 2007. The thick line gives the mean of the projection, the two transitions from dark to light shaded areas the 25th and 75th percentile and the boundaries the 5th and 95th percentile of the distribution. The graph states e.g. that the debt-to-GDP ratio is expected to be between 52 and 63 percent with a probability of 50% in its stationary distribution or between 56 and 61 percent in 2009 with the same probability. From the figure, it is obvious that the rapid increase in the debt-to-GDP ratio exceeding 80% in 2014 marks a tail event coming as a deep surprise from pre-crisis perspectives with a probability far below 5% at the time. The figure further shows that the historical record of Austrian fiscal policy before the crisis suggests a mean stabilization of the debt-to-GDP ratio slightly below the 60% Maastricht threshold. The probability of being below 62% in the stationary state was about 75% and the probability of being below 68% about 95% such that one can consider with confidence that Austrian fiscal policy was in line with the Maastricht treaty before the crisis and that expectations of ever rising debt were not supported by underlying fundamentals.

The period following the financial crisis of 2007 is characterized by an abrupt increase in sovereign debt while the impact on budget balances is mitigated by low real interest rates enhancing the ability of the government to serve its debt obligations. Shall one expect that the return back to normal refinancing conditions will cause public deficits to explode as a result of overwhelming costs of debt servicing? Figure (3) shows that this view is also misguided. Starting from the actual position in 2014, the debt-to-GDP ratio exhibits a strong mean reverting tendency back to its stationary distribution over the next decade with mean close to the 60% criterion and 95th percentile modestly above the threshhold. From the point of view of rational investors, the unexpected increase in sovereign debt obligations induced by the "Great Recession" does not motivate worries

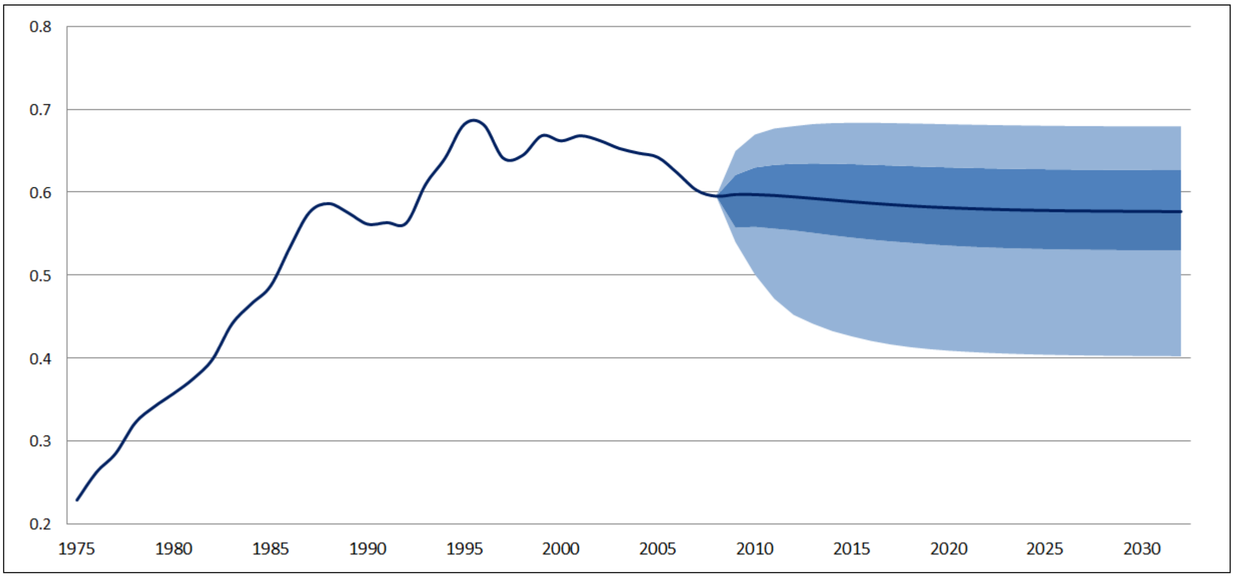

Figure 2: Starting from 2007 values

about an endless upward spiral of debt. The modest risk premiums paid on Austrian sovereign bonds do therefore not reflect the expectation of substantial changes in fiscal policy, but rather the continuation of the historical record. This record exhibits a strong tendency to lean against rising debt reflected by the comparatively high value of $\alpha = 0.44$ which is substantially above the result of Haber and Neck (2006) who estimate a smaller fiscal reaction parameter of 0.09 between 1960 and 2003 by applying the Bohn-Test. Furthermore, while leaning against debt is partially forced from outside on Eurozone member countries due to the treaty of Maastricht, it is however unclear whether the treaty will succeed to ensure high levels of fiscal reaction to rising debt in future. A strong reaction of Austrian fiscal authorities can therefore considered likely, but not ensured. From these considerations, a sensitivity analysis makes sense.

Figure (4) indicates that choosing the more moderate value of $\alpha = 0.09$ solely reduces the convergence speed of the debt-to-GDP ratio towards its stationary equilibrium while the implied long-run distribution is hardly affected. Figure (5) shows the projection starting in 2014 with α set to zero, i.e. the extreme case that future governments will not react to changes in the public debt position. The graph illustrates that such a fiscal policy generates a small likelihood of public debt to exceed 120% of GDP in the long

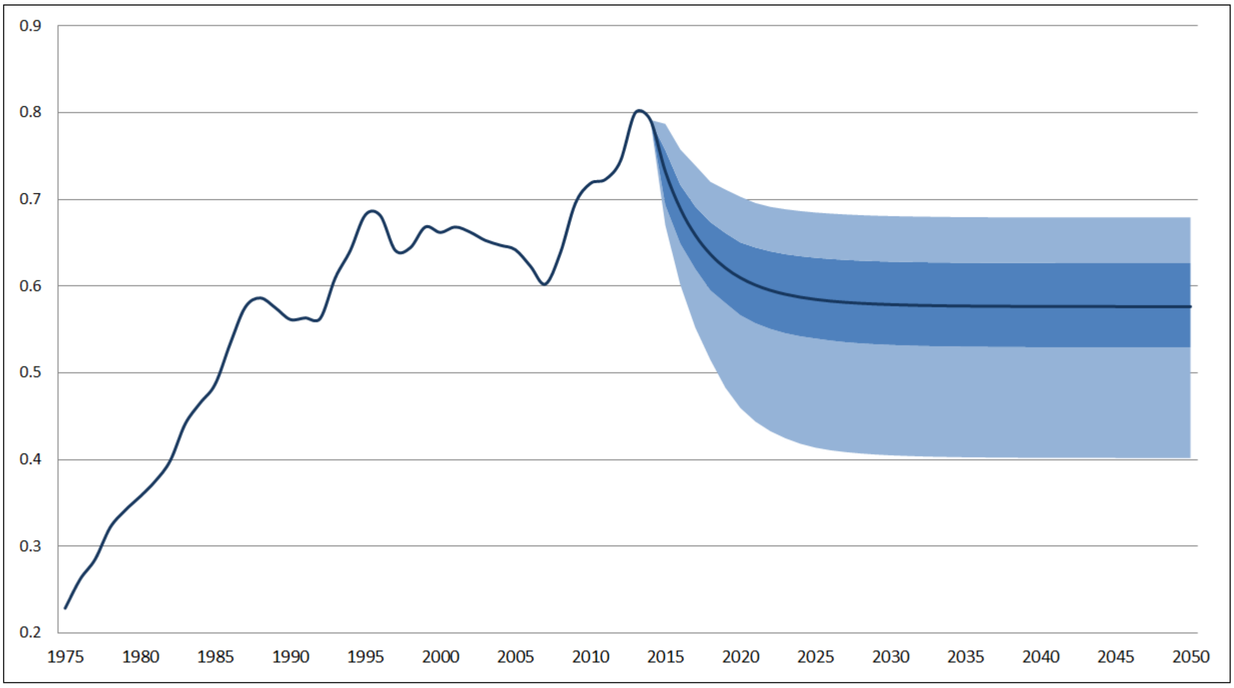

Figure 3: Main projection with $\alpha = 0.44$

run yet also a much stronger probability of debt to fall below 20% of GDP or even turn negative. While it is unclear from public finance theory that such potentially low levels of sovereign debt are welfare optimizing, the important result from figure (5) is that even without future leaning against debt, the debt-to-GDP ratio is not supposed to follow an explosive path thus potentially triggering (self-fulfilling) fears of default. Consequently, even if investors assume for any reason that the Austrian government will fail entirely to lean against debt in future, there is no reason to question the solvency of the public sector based on the historical trajectory of primary deficits and surpluses. While very low levels of α are not justified by past reactions to debt, the above argument underlines the stability of Austria's fiscal position.

This conclusion exhibits yet an important caveat - it incurs the assumption that the state dependent primary balance remains constant over time and thus abstracts from the potentially large burden induced by the costs of ageing over the coming decades. European-Commission (2012) quantifies those costs by 4.4% of GDP until 2060. Figure (6) shows that increasing the exogenous component of the primary deficit by the much

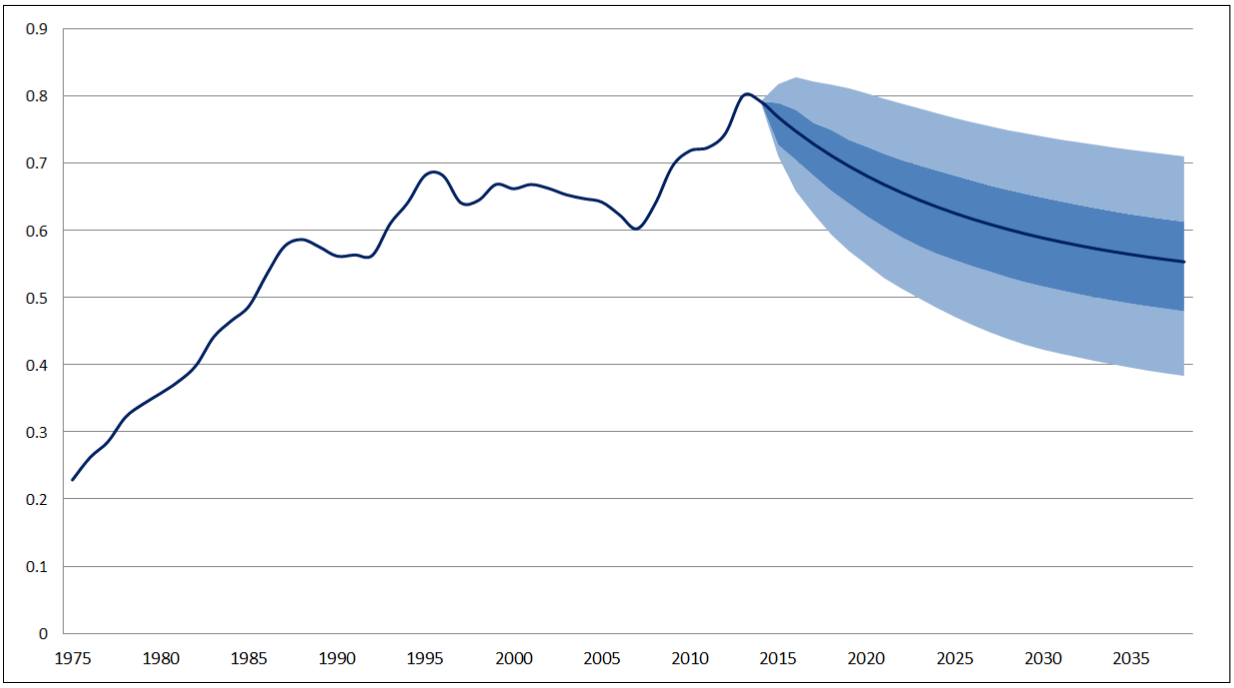

Figure 4: Projection with $\alpha = 0.09$

smaller value of 0.01 while remaining $\alpha = 0$ already causes a considerable probability of debt to exceed Italian levels thus incurring a likelihood of debt crisis. This result is magnified by stronger effects of ageing on the primary balance. Low yields on 30-year-bonds reflect investor confidence that the government will offset the effect of ageing on the primary balance by a combination of spending cuts and/or increased tax earnings. Needless to say that appropriate fiscal measures may not negatively affect economic growth in order for the previous argumentation to remain valid.

Figure (7) finally compares our long-run projection with the results of Schimann (2013) reflecting the long-run projections of the Austrian Institute of Economic Research (WIFO) which are based on a Cobb-Douglas production function with exogenously given technical progress. The long-run projection closely mimics the mean of the stationary distribution despite the differences in the employed methods.

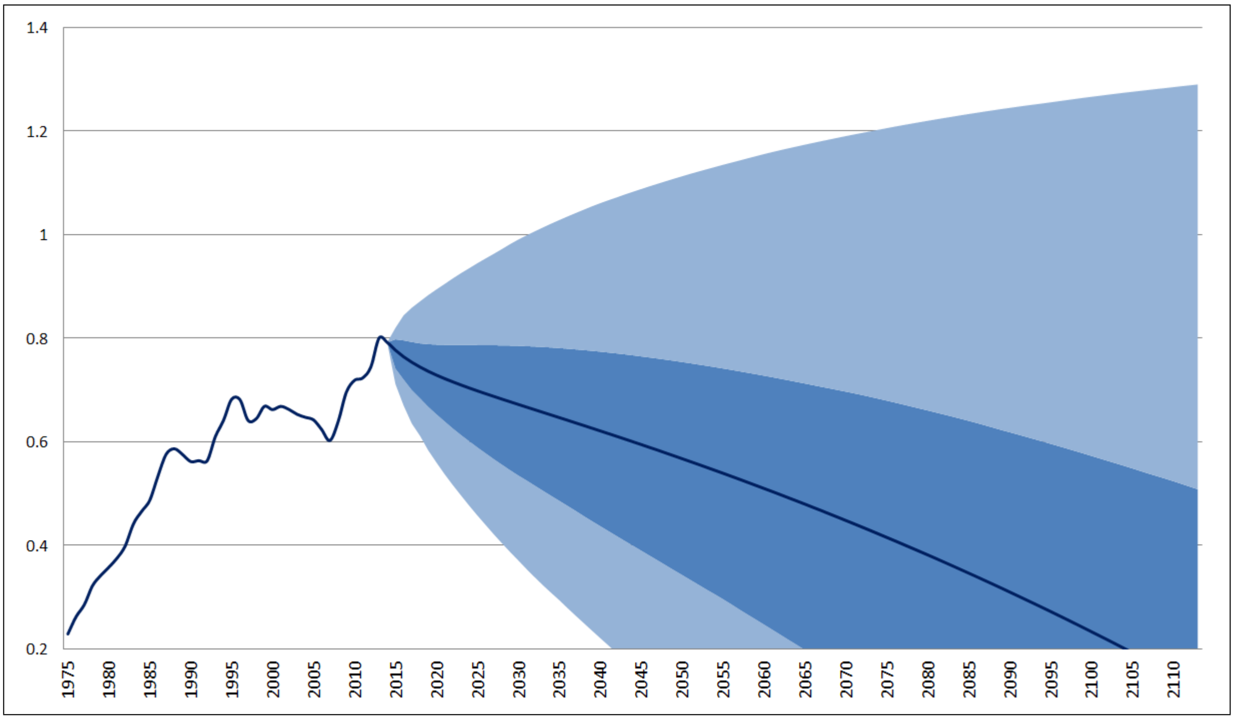

Figure 5: Projection with $\alpha = 0$

6. Conclusion

Using a Bayesian estimator to construct the transition probabilities of fundamental states meaningfully extends the method used in Hall (2014). Our showcase application to the stability of Austrian public finances demonstrates that the increase in sovereign debt after 2007 represents an unexpected and transitory shock induced by the financial crisis while Austrian fiscal policy was roughly in line with the 60% Maastricht threshold over the last decades. Fears of an endless upward spiral of debt strengthened by potential increases in world interest rates are not supported by the historical record, a result which even holds in the unlikely case that the government fails entirely to lean against debt in future. Offsetting the cost of ageing is necessary for this conclusion to hold and prevent debt explosion. Our baseline projection is closely in line with Schimann (2013) despite the employment of a fundamentally different approach underlining the validity of the method and the results of both attempts.

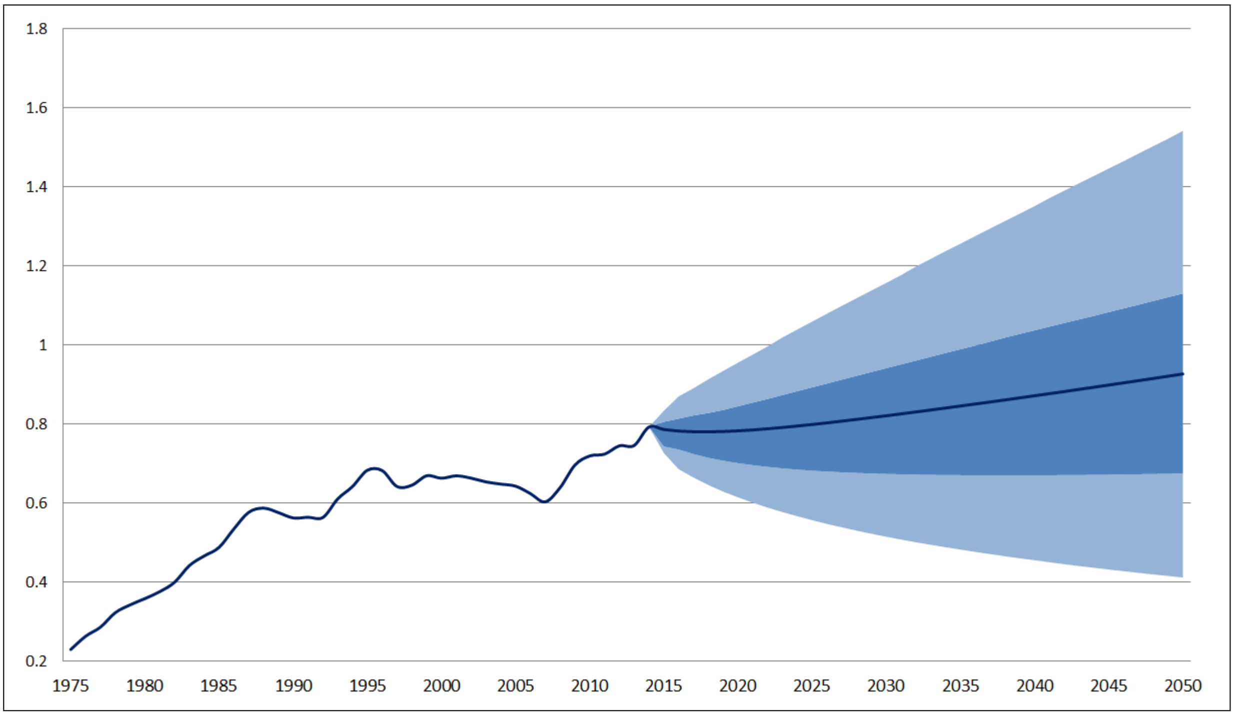

Figure 6: Projection with $\alpha = 0,\ xcon = 0.01$

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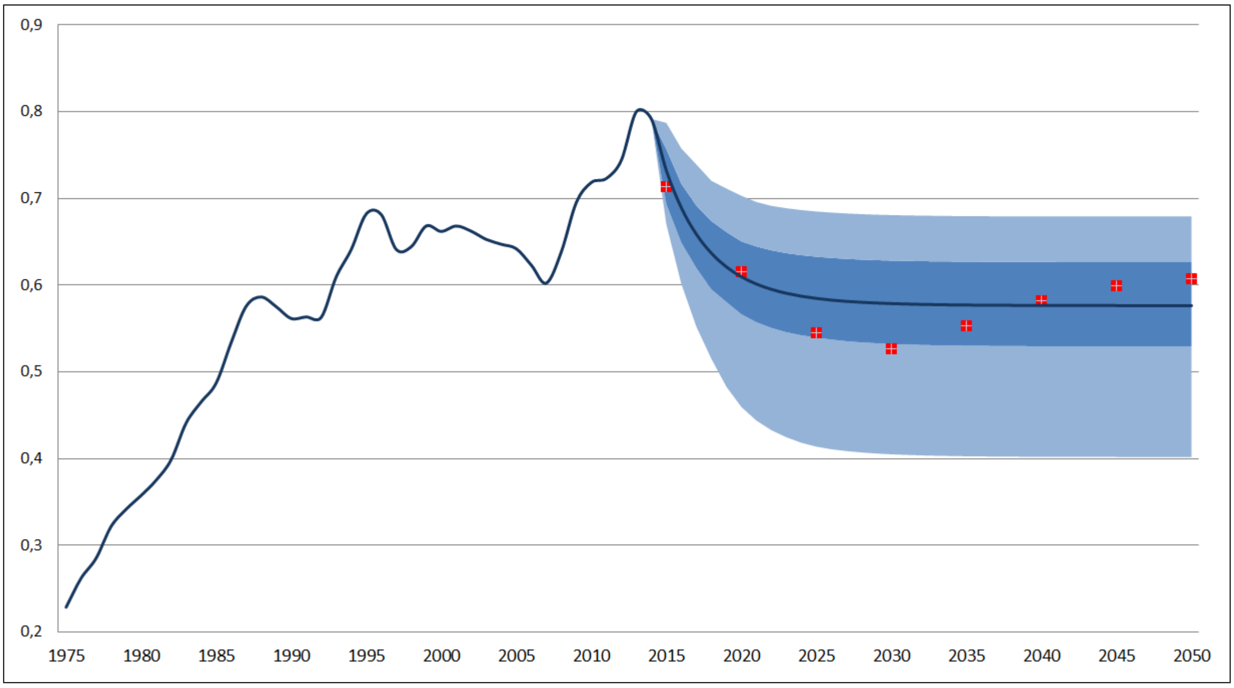

Figure 7: Comparison to Wifo Forecast

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Appendix

This appendix describes the data used in the empirical application.

Variable	Description	Source
g	Growth rate of real GDP	1975-2011: IMF "Public Finances in Modern History" Database (Mauro et al. (2013)) 2012-13: IMF World Economic Outlook
π	CPI inflation rate	1975-2010: Reinhart and Rogoff (2011) 2011-13: IMF WEO
u	Unemployment rate	1975-2013: OECD Annual Labour Force Statistics
r	Long-term real interest rate	1975-2011: Mauro et al. (2013) 2012-13: IMF WEO
d	Government primary deficit, percent of GDP	1975-2011: Mauro et al. (2013) 2012-13: IMF WEO
b	Gross government debt, percent of GDP	1975-2011: Mauro et al. (2013) 2012-13: IMF WEO

Table 2: Data description and sources

Year	g	π	u	r	d	b	s
1975	-0.36	8.40	1.8	1.21	1.42	22.9	2
1976	4.58	7.30	1.8	1.45	1.99	26.2	2
1977	5.01	5.50	1.6	3.24	0.33	28.5	2
1978	-0.15	3.60	2.1	4.61	0.55	32.2	1
1979	5.46	3.71	2.1	4.26	0.09	34.1	4
1980	1.78	6.33	1.9	2.92	-0.77	35.7	2
1981	-0.14	6.81	2.5	3.80	-0.87	37.5	2
1982	1.95	5.44	3.5	4.48	0.32	39.9	2
1983	2.95	3.34	4.1	4.84	1.32	44.1	4
1984	0.06	5.67	3.8	2.35	-0.58	46.5	2
1985	2.46	3.19	3.6	4.58	-0.72	48.7	4
1986	2.32	1.74	3.1	5.59	0.32	53.4	4
1987	1.35	1.41	3.8	5.50	0.52	57.6	4
1988	2.87	1.87	3.6	4.99	0.12	58.6	4
1989	3.74	2.25	3.1	4.90	-0.23	57.5	4
1990	4.17	2.76	3.2	6.00	-0.89	56.1	4
1991	3.34	3.12	3.5	5.44	-0.69	56.3	4
1992	1.89	3.43	3.6	4.75	-1.73	56.3	4
1993	0.37	3.24	4.2	3.47	0.53	60.9	1
1994	2.21	2.71	4.4	4.32	1.21	64.1	4
1995	2.54	1.60	3.9	5.54	1.80	68.2	4
1996	2.23	1.79	4.3	4.53	0.13	68.1	4
1997	2.13	1.16	4.4	4.52	-1.79	64.1	5
1998	3.60	0.82	4.5	3.89	-1.22	64.4	5
1999	3.34	0.52	3.9	3.58	-1.08	66.8	5
2000	3.65	1.96	3.6	3.63	-1.76	66.2	5
2001	0.52	2.30	3.6	2.88	-3.32	66.8	1
2002	1.65	1.70	4.2	3.37	-2.42	66.2	5
2003	0.80	1.30	4.3	2.91	-1.42	65.3	1
2004	2.54	1.95	5.0	2.03	1.61	64.7	5
2005	2.46	2.11	5.2	1.06	-1.18	64.2	3
2006	3.60	1.69	4.8	2.32	-1.19	62.3	5
2007	3.73	2.20	4.4	2.08	-1.88	60.2	5
2008	2.18	3.22	3.8	1.11	-1.66	63.8	3
2009	-3.81	0.40	4.8	3.41	1.34	69.5	1
2010	2.31	1.50	4.4	1.39	1.74	71.8	3
2011	3.11	3.55	4.1	0.05	0.44	72.3	3
2012	0.87	2.58	4.3	-0.21	0.32	74.4	3
2013	0.35	2.12	4.9	-0.11	-0.16	74.5	3

Table 3: Data and fundamental states for Austria

		s'				
		1	2	3	4	5
	1	0	0	0.50	0.50	0
	2	0	0.29	0.29	0.29	0.14
s	3	0.08	0.08	0.62	0.08	0.15
	4	0	0.40	0.20	0.20	0.20
	5	0.13	0.13	0.25	0	0.50

Table 4: Maximum likelihood estimates of transition probabilities

		s'				
		1	2	3	4	5
	1	0.12	0.04	0.21	0.23	0.39
	2	0.12	0.61	0.06	0.19	0.02
s	3	0.06	0.01	0.77	0.05	0.10
	4	0.05	0.11	0.02	0.76	0.05
	5	0.17	0.04	0.15	0.00	0.64

Table 5: Posterior mean estimates of transition probabilities