A Sector-Specific Methodology for Evaluating Industrial Dynamics

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Abstract

The present study deals with a methodology combining properties of Input-Output approaches and computable general equilibrium models in order to analyse and evaluate industrial dynamics. Symmetrical Input-Output tables are used to fully specify production, distribution and consumption of an economy where factor substitution is allowed for, which is a main difference to Leontief models. The parameters determined are dimensionless and allow for international and intertemporal comparisons. A case study of Austria and the USA shows that there is convergence of some industries while in other sectors divergence tendencies can be observed. Technical change does not affect all countries uniformly or unidirecticonally.

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1 Introduction

The aim of this paper is to present a new methodology for evaluating structural change and industrial dynamics. It combines features of Input-Output analysis and neoclassical general equilibrium theory in order to provide a better understanding of the fundamentals of the underlying economic structures. The approach at hand allows for many applications that are not yet all explored in detail and opens a new future research agenda. The methodology will be applied for an intertemporal and international comparison of the economies of Austria and the United States of of America.

The theoretical background of the new methodology is the combination of aspects of Piero Sraffa's masterpiece *Production of Commodities by Means of Commodities* (Sraffa 1960), the *Input-Output analysis* of Wassily Leontief (1953) and neoclassical *General Equilibrium Theory* elaborated by Arrow & Debreu (1954). One basic difference of these approaches is the possibility of substitution between inputs, that ranges from elasticities of substitution from 0 (Leontief production function) to 1 (Cobb-Douglas function). Another notion of substitutability in terms of choice of technique was introduced in Leontief and Sraffa models to overcome this shortcoming. The present study takes the data set of symmetric Input-Output tables and uses it for the formulation of a neoclassical general equilibrium. The merit of this approach is the simple structure of the model, the easy identification of parameters from statistical data that are usually available and the comprehensive interpretation of the parameters. Cobb-Douglas functions are basically specified by elasticities that are dimensionless variables in contrast to Input-Output coefficients that either depend on measurement units or prices. Elasticities always have the same meaning, in intertemporal as well as in international comparisons.

The innovation of the present approach is the introduction of production of commodities by means of commodities in the Cobb-Douglas function. Typically, neoclassical production functions treat "capital" and labour as inputs, which turns out to be difficult to sustain. Fisher (1969) already showed the shortcomings of this idea in the 1960s, but the discussion moved on. Labini (1995) urged for a change in the interpretation of the Cobb-Douglas production function and despite several criticims, Balistreri et al. (2003) argues in favour of it.

The methodology is applied to Austria and the USA for the years 1995 and 2000 in order to identify structural changes in the sectors of the economy. The rich database of Input-Output tables of the OECD allows one to identify 38 different sectors and gives a detailed view on the branches of both economies. A comparison of the factor elasticities between Austria and the USA provides an insight into technological differences and similarities of production structures. Furthermore, gaps in labour productivity can be identified and different consumer preferences are easily discovered. The evolution of the parameters from 1995 to 2000 allows one to account for technological progress and industrial dynamics. Productivity gains are identified and changes in the distribution and consumption patterns are observed.

The report is organized in the following way. Chapter 2 elaborates the theoretical framework of the methodology used. In the first section, it will be discussed why neoclassical frameworks sometimes have problems dealing with production of commodities by means of commodities. The next section shows how statistical data from Input-Output tables are used to fully identify a general equilibrium of a Cobb-Douglas economy. Merits and demerits of this approach will be presented. Chapter 3 shows the implementation of the theoretical framework. The methodology is applied to Austria and the United States of America and structural changes within each country as well as international similarities and differences are identified. The general equilibrium approach developed in this study is also compared to traditional Input-Output modelling. Concluding remarks can be found in chapter 4 that summarizes the results and deals with the implications for structural change and industrial dynamics.

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2 Theoretical Notes

This chapter deals with two theoretical aspects of the intersection of neoclassical and neo-Ricardian economics. Section 2.1 analyses the problem of neoclassical economics incorporating production of commodities by means of commodities. It shows that a positive elasticity of substitution may not be compatible with constant returns to scale. Section 2.2, "Symmetric Input-Output Tables as General Equilibria", provides a simple method of implementing statistical data into a neoclasscial computable general equilibrium (CGE) setting. It gives rise to a new interpretation of Input-Output (IO) coefficients as factor elasticities of a Cobb-Douglas function.

2.1 Production of Commodities by Means of Commodities

and Why Neoclassical Theory has Troubles Explaining it

Let us consider a simple n-sectoral model in a Cobb-Douglas economy with constant returns to scale. The notions of sector and industry are used synonymously in this paper. The terms good and commodity are considered equal and also comprise services. It is assumed, that each sector/industry produces exactly one good and joint production is ruled out. Furthermore, no fixed capital is required in the process of production. Companies maximize their profits with respect to given prices. The output of industry j, $j = 1, ..., n$, will be labeled x_j and the input of good i in the production of good j shall be x_{ij}. The uniform price of commodity j is p_j. The production function is given by

$$x_j = \alpha_j \prod_{i=1}^{n} x_{ij}^{\alpha_{ij}} \text{ with } \sum_{i=1}^{n} \alpha_{ij} = 1, \alpha_j > 0 \; \forall j \tag{2.1}$$

For simplicity we assume that all inputs are required: $\alpha_{ij} > 0$, $\forall$ i,j[1]. All factors are required at the beginning of the period and so r is the uniform interest rate. $\alpha_j > 0$ is a shifting or efficiency parameter. Profit maximization in sector j requires

$$\max_{x_{ij}} \left(x_j \frac{p_j}{1+r} - \sum_{i=1}^{n} x_{ij} p_i \right) \Rightarrow \max_{x_{ij}} \left(\alpha_j \prod_{i=1}^{n} x_{ij}^{\alpha_{ij}} \frac{p_j}{1+r} - \sum_{i=1}^{n} x_{ij} p_i \right) \tag{2.2}$$

[1] In case of $\alpha_{ij} = 0$ for some inputs, we must define expressions like $\alpha_{ij}^{-\alpha_{ij}}$ to be 1 for these inputs. The results remain the same ones.

The discounted value of the output minues input cost is to be maximized by chosing appropriate input combinations. The first order conditions resulting from equation (2.2) are ovbiously the following well known marginal product rules.

$$MP_{ij}\frac{p_j}{1+r} - p_i = 0 \quad \forall i, \text{ with } MP_{ij} = \frac{\partial x_j}{\partial x_{ij}} \tag{2.3}$$

The interesting point in the case of production of commodities by means of commodities is that good j also enters into its own production. For the input x_{jj} the marginal product rule of equation (2.3) reduces to $MP_{jj} = 1 + r$. An additional marginal unit of this input must also increase the output by one marginal unit times the interest factor $1 + r$.

A further step can be made if we consider the marginal product in the Cobb-Douglas case, which can easily be expressed explicitly.

$$MP_{ij} = \alpha_{ij}\frac{x_j}{x_{ij}} \quad \forall i, j \tag{2.4}$$

For MP_{jj} we can thus derive the following result.

$$MP_{jj} = \alpha_{jj}\frac{x_j}{x_{jj}} = 1 + r \Rightarrow x^*_{jj} = \alpha_{jj}\frac{x_j}{1+r} \tag{2.5}$$

An asterisk denotes an optimal value. (2.5) shows that the optimal factor demand is independent of the price for the good entering its own production. The optimal factor demands of all other products can be derived by using (2.3) and (2.4) in the production function.

$$x^*_{ij} = \frac{x_j}{\alpha_j}\frac{\alpha_{ij}}{p_i}\prod_{k=1}^{n}\left(\frac{p_k}{\alpha_{kj}}\right)^{\alpha_{kj}} \tag{2.6}$$

Factor demands obviously depend on input prices, since the Cobb-Douglas production function allows for factor substitution. Due to constant returns to scale, factor proportions do not vary if the output level x_j changes. Some mainpulations of equation (2.6) show an even more simple formula for optimal factor demands.

$$x^*_{ij} = \alpha_{ij}x_j\frac{p_j}{p_i\,(1+r)} \tag{2.7}$$

So far, these results are not yet surprising. Using the optimal input of x_{jj} in (2.6) yields an interesting result.

$$\alpha_{jj}\frac{x_j}{1+r} = \frac{x_j}{\alpha_j}\frac{\alpha_{jj}}{p_j}\prod_{k=1}^{n}\left(\frac{p_k}{\alpha_{kj}}\right)^{\alpha_{kj}} \Rightarrow p_j = \frac{1+r}{\alpha_j}\prod_{k=1}^{n}\left(\frac{p_k}{\alpha_{kj}}\right)^{\alpha_{kj}} \quad , \forall j \tag{2.8}$$

The price system is independent of the quantities produced, such that the levels of procution have no influence on the price system. (2.8) consists of n equations and given all technological parameters α_{ij} and α_j, $n-1$ relative prices as well as the interest rate r can be calculated in general. (2.8) is the equivalent to the price system of Sraffa (1960) without labour inputs or a zero wage rate. **A** is a coefficient matrix and the price equation is given in obvious notation.

$$\mathbf{A}\mathbf{p}\,(1+r) = \mathbf{p} \tag{2.9}$$

$1+r$ will be equal to the inverse of the maximum Eigenwert of **A** and **p** will be the corresponding Eigenvector. The solution of the nonlinear equations in (2.8) and the determination of prices and the profit rate is usually harder than in the linear case.

The problem of many neoclassical models is the disregard of the time structure. Typically, instantaneous productions is assumed. In the model at hand, this corresponds to applying a discounting factor of 1 and thus setting the interest rate ex ante at 0. In terms of the relative prices given in equation (2.8), the system is overdetermined and in nontrivial cases no solution will exist. This relationship can be shown in a very simple two sectoral model. Optimal factor demands for x_{11} and x_{22} imply according to (2.8) the following price system.

$$p_1 = \left(\frac{p_1}{\alpha_{11}}\right)^{\alpha_{11}} \left(\frac{p_2}{\alpha_{21}}\right)^{\alpha_{21}} \left(\frac{1+r}{\alpha_1}\right) \Rightarrow p_1 = \alpha_{11}^{\frac{-\alpha_{11}}{\alpha_{21}}} \alpha_{21}^{-1} p_2 \left(\frac{1+r}{\alpha_1}\right)^{\frac{1}{\alpha_{21}}} \tag{2.10}$$

$$p_2 = \left(\frac{p_1}{\alpha_{12}}\right)^{\alpha_{12}} \left(\frac{p_2}{\alpha_{22}}\right)^{\alpha_{22}} \left(\frac{1+r}{\alpha_2}\right) \Rightarrow p_2 = \alpha_{12}^{\frac{-\alpha_{12}}{\alpha_{22}}} \alpha_{22}^{-1} p_1 \left(\frac{1+r}{\alpha_2}\right)^{\frac{1}{\alpha_{22}}} \tag{2.11}$$

These equations imply a formula for the interest factor compatible with perfect competition in these two industries.

$$1+r = \left[\alpha_1^{\frac{1}{\alpha_{21}}} \alpha_2^{\frac{1}{\alpha_{22}}} \alpha_{11}^{\frac{\alpha_{11}}{\alpha_{21}}} \alpha_{12}^{\frac{\alpha_{12}}{\alpha_{22}}} \alpha_{21}\alpha_{22}\right]^{\frac{\alpha_{21}\alpha_{22}}{\alpha_{21}+\alpha_{22}}} \tag{2.12}$$

In a viable economy, the interest rate r will be positive. The neoclassical assumption of instantaneous production and a zero interest rate will typically not hold with production of commodities by means of commodities.

It is easy to construct paradoxa for the n-dimensional case as well. If we consider n-sectoral production and we assume production technologies with $\alpha_{ij} = 1/n\ \forall i,j$ and $\alpha_j = \alpha\ \forall j$, then price equation (2.8) reduces to $p_j/\left(1+r\right) = \alpha n p_j,\ \forall j$. For $r=0$ this is only true in the trivial case of $\alpha = n$.

This result is not limited to the case of the Cobb-Douglas function, it also affects the more general Constant Elascticity of Subsitution (CES) function. The properties of this type of functions are extensively discussed in Arrow et al. (1961). With constant returns to scale, we have:

$$x_j = \alpha_j \left(\sum_{i=1}^{n} \alpha_{ij}^{\frac{1}{\sigma_j}} x_{ij}^{\frac{\sigma_j-1}{\sigma_j}}\right)^{\frac{\sigma_j}{\sigma_j-1}} \tag{2.13}$$

The parameter σ_j denotes the elasticity of substitution and the Cobb-Douglas production function is obtained in the limiting case of σ_j going to unity. The marginal product of factor x_{ij} of a CES production function can be expressed in a similar manner as in the case of a Cobb-Douglas function.

$$MP_{ij} = \left(\alpha_{ij} \frac{x_j}{x_{ij}} \right)^{\frac{1}{\sigma_j}} \tag{2.14}$$

In case of $\sigma_j = 1$, the CES function becomes the Cobb-Douglas function and the exponent is one. For a commodity x_{jj} used in its own production, the marginal physical product must equal 1+r and thus a similar relation as in the Cobb-Douglas case applies: $x^*_{jj} = \alpha_{jj} x_j / (1+r)^{\sigma_j}$. The profit maximizing factor demand for any factor is given by the next equation.

$$x^*_{ij} = \alpha_{ij} x_j \left(\frac{p_j}{p_i (1+r)} \right)^{\sigma_j} \tag{2.15}$$

Using equation (2.15) in the production function constrains the price system in sector j of the economy in the following way.

$$p_j = \frac{(1+r)}{(\alpha_j)^{\sigma_j}} \left(\sum_{k=1}^{n} \alpha_{kj} p_k^{1-\sigma_j} \right)^{\frac{1}{1-\sigma_j}} \tag{2.16}$$

Given one constraint for each one of the j sectors, these restrictions will overdetermine the price system in case of $r = 0$. This shows that in any CES production system with constant returns to scale, where commodities are produced by means of commodities, a neoclassical general equilibrium with instantaneous production usually leads to paradoxa. Any approach has to work either with decreasing returns to scale[2], exclude substituability like the Leontief model, or take into account that production takes time.

Many models still rely on aggregate production functions with capital and labour as inputs. This approach has been debated in the scientific community. Fisher (1969) already pointed out that the theoretical preconditions for the existence of such an aggregate production function are hardly given and the article of Felipe & Adams (2005) shows that the parameters of a Cobb-Douglas function with capital and labour inputs are merely the outcome of an income distribution identity.

A common approach used to incorporate intermediate products in CGE modelling is to impose nested CES functions. Examples can be found, for instance in Zhang (2008, p.10), who uses a three tier model. On the first tier, output is produced according to a Leontief technology with respect to intermediate products and value added. On the second tier, value added is determined via a CES function according to the input of aggregate capital and labour.

[2] Increasing returns to scale are in general not compatible with a competitive equilibrium.

Finally, capital inputs are chosen with the help of a Leontief function. Such approaches are also presented in Shoven & Whalley (1992) and they are the most frequently used ones in CGE models. The foundation of this method was elaborated by Armington (1969). Nevertheless, this way of dealing with intermediate products and circular production appears to be rather artificial and more complicated than the approach presented in the next section.

2.2 Symmetric Input-Output Tables as General Equilibria

In this section, it will be shown that the data from an Input-Output table can be interpreted as a neoclassical general equilibrium with substitution among inputs. There exists an extensive literature on general equilibrium models and especially on computable general equilibrium (CGE) models. Since the emergence of powerful computing devices in the 1980s, especially applied approaches and empirical studies have become more and more popular (Scarf & Shoven 1984, Bergman et al. 1990, Shoven & Whalley 1992, Santis 1996, Fossati 1996). Today, there are even special software packages devoted to CGE modeling as Rutherford (1999) points out. Theoretical backgrounds and extensions can also be found in various books and papers (Balasko 1988, Dinwiddy & Teal 1988, Codenotti 2005, Jain & Varadarajany 2006).

The economy at hand will be characterized by a set of production technologies described by a Cobb-Douglas function for each sector. Households' preferences are also modelled with a Cobb-Douglas function. The only primary factor in this economy is labour and it is assumed that the labour supply of households is completely inelastic with respect to the wage rate.[3]

The production side of the economy is given by n Cobb-Douglas functions, one for producing each of the intermediate and consumption goods x_j, $j = 1, ..., n$. The factors of production are these n commodities and the primary input labor ℓ. For notational simplicity, labour will also be denoted by x_{n+1}, especially when sums or products are involved. The production technology of good x_j, $j = 1, ..., n$ is given by:

$$x_j = f_j\left(x_{1j}, ..., x_{n+1,j}\right) = \alpha_j \prod_{i=1}^{n+1} x_{ij}^{\alpha_{ij}} \qquad \text{with } \alpha_{ij} > 0, \alpha_j > 0 \;\forall i, j \tag{2.17}$$

All goods are *basics* and the production technology is assumed to be strictly concave. Increasing returns to scale shall be excluded because they are usually not compatible with price taking behaviour. Furthermore, constant returns will not be treated in this analysis according to the arguments given in chapter 2.1. The exponents of the Cobb-Douglas function in (2.17) are therefore subject to the following restriction.

[3]It would be possible, to endogenize the labour supply by incorporating the disutility of work into the utility function. This approach would, however, be artificial and not add any substantial information to the analysis.

$$\sum_{i=1}^{n+1} \alpha_{ij} < 1, \forall j \tag{2.18}$$

The rationale of this assumption and its merit will be explained in greater detail when the factor demand of all sectors will be derived. There is exactly one production function for each good and it shall be assumed that the production function describes the technological possibilities of the entire sector. A sector is thus treated as a single company.

In a next step, the household sector will be described. Households are assumed to maximize their utility u for a given income and given prices $p_1, ..., p_n$ of the consumption goods $c_1, ..., c_n$. All goods produced are, at least in small amounts, consumption goods. Household utility is modelled with the following Cobb-Douglas function:

$$u = g(c_1, ..., c_n) = \prod_{j=1}^{n} c_j^{\beta_j} \qquad \text{with } \beta_j > 0, \forall j \tag{2.19}$$

Households are endowed with $\bar{\ell}$ units of labour. The supply of labour to the company sector is inelastic, so $\bar{\ell}$ will be supplied regardless of the remuneration. We will use the convention $\bar{\ell} = \bar{x}_{n+1}$. In the following, exogenously given data will be marked by a bar.

In addition to the households, there is also a further sector consuming final goods. In this simplified model, it is assumed that its demand is exogenous. In terms of national accounts, this sector corresponds in principle to government consumption, fixed capital formation and net exports. There are also possibilities to endogenize this final demand, which might be the subject of a future research. The exogenously given consumption of good j will be labeled $\bar{e_j}$.

2.2.1 Profit and Utility Maximization

We assume that all economic agents take prices as given, there are no market failures, no uncertainty or any other circumstances that are not compatible with a competitive equilibrium.

In the following optimization problems, it will be useful to use matrix notation. Unless indicated otherwise, all vectors introduced will be column vectors. $\mathbf{x}$ is the vector consisting of the output of the n different production goods: $\mathbf{x} = (x_1, ..., x_n)^{\mathbf{T}}$. In a similar manner, $\mathbf{c}$ and $\bar{\mathbf{e}}$ denote the column vectors of consumption and exogenous demand. The row vector $\mathbf{p}$ denotes the vector of the n commodity prices and w denotes the wage rate, that is the price of labour. For reasons of notational simplicity, we define $w = p_{n+1}$.

Company j producing good j seeks to maximize its profits π_j for given prices $p_1, ..., p_{n+1}$. In the short run, profit rates are not necessarily uniform in an economy and extra profits may exist, so no discounting factor is used in the following optimization problem.

$$\max_{x_1, ..., x_{n+1}} \pi_j = p_j \alpha_j \prod_{i=1}^{n+1} x_{ij}^{\alpha_{ij}} - \sum_{i=1}^{n+1} x_{ij} p_i, \forall j \tag{2.20}$$

The maximization program is concave and the first order conditions are necessary and sufficient for an optimum. This optimization problem leads to the well known marginal conditions that imply that the marginal value product of each factor must equal its price in equilibrium.

Some algebraic manipulations of these results lead to the optimal factor demands for a given wage rate and prices. In the following, optimal variables will be denoted with an asterisk.

$$x_{ij}^{*} = \frac{\alpha_{ij}}{p_i} \left(p_j \alpha_j\right)^{\frac{1}{1-s_j}} \prod_{i=1}^{n+1} \left(\frac{\alpha_{ij}}{p_i}\right)^{\frac{\alpha_{ij}}{1-s_j}}, \forall j \tag{2.21}$$

In equation (2.21), s_j denotes the degree of homogeneity of the production function for commodity j, which equals the sum of all exponents α_{ij}.

Due to decreasing returns to scale, the optimal factor inputs do not depend on the production levels and are uniquely determined by prices and the wage. If there were constant or increasing returns to scale, different properties would hold. With constant returns to scale, companies would make zero extra profits and the price would equal the average cost in equilibrium. The scale of production would be indeterminate. Increasing returns to scale would usually not be compatible with price taking behaviour and a competitive equilibrium.

Only in the case of decreasing returns to scale, sector j's output is uniquely determined by prices and the wage rate.

$$x_j^{*} = p_j^{\frac{s}{1-s_j}} \alpha_j^{\frac{1}{1-s}} \prod_{i=1}^{n+1} \left(\frac{\alpha_{ij}}{p_i}\right)^{\frac{\alpha_{ij}}{1-s_j}}, \forall j \tag{2.22}$$

$$\pi_j^{*} = \left(1-s_j\right) \left(p_j \alpha_j\right)^{\frac{1}{1-s}} \prod_{i=1}^{n+1} \left(\frac{\alpha_{ij}}{p_i}\right)^{\frac{\alpha_{ij}}{1-s_j}}, \forall j \tag{2.23}$$

As it can easily be shown, optimal factor and labour demands and the optimal output are homogeneous of degree zero with respect to input prices and the wage rate. Profit is therefore homogeneous of degree one in all prices.

The optimization problem of the household sector is given by:

$$\max_{\mathbf{c}} \; u = \prod_{j=1}^{n} c_j^{\beta_j} \tag{2.24}$$

$$\text{s.t.} \qquad \mathbf{pc} = y \tag{2.25}$$

The utility is maximized subject to the budget constraint, where y equals the disposable income of the household:

$$y = w\bar{\ell} + \sum_{j=1}^{n} \pi_j - \mathbf{p\bar{e}} \tag{2.26}$$

The consumption expenditures $\mathbf{pc}$ must equal the disposable income, which consists of the wage income $w\bar{\ell}$ plus the profits of all companies less the expenses for exogenous expenditures $\mathbf{p\bar{e}}$. These expenditures could be interpreted as a tax that is used in order to finance the exogenous demand. The household sector is a price taker and a change in their decision variables is taken to have no effect on their income.

The optimal consumption level of good i depends on prices and disposable income:

$$c_i^* = \beta_i \frac{y}{p_i}, \, \forall i \tag{2.27}$$

Optimal consumption is homogeneous of degree zero and thus depends on relative prices only. Absolute price levels do not matter. Slightly rearranged, equation (2.27) can also be interpreted in a very intuitive way.

$$\frac{c_i^* p_i}{y} = \beta_i, \, \forall i \tag{2.28}$$

The value share of good i in the consumption bundle equals the exponent β_i, which is a very convenient property of the Cobb-Douglas production function.[4]

2.2.2 Market Clearing and Equilibrium

The economy at hand is given by a set of production functions for each sector, a utility function and a fixed labour endowment $\bar{\ell}$ for the households and a vector of exogenous expenses $\bar{\mathbf{e}}$. Market clearing in such an economy implies that the following equations are satisfied.

$$\sum_{i=1}^{n} x_{ij} + c_j + \bar{e}_j = x_j, \, \forall j \tag{2.29}$$

$$\sum_{i=1}^{n} \ell_j = \bar{\ell} \tag{2.30}$$

Markets for the n different intermediate and consumption goods and the labour market must clear simultaneously. In an equilibrium with profit and utility maximization, the optimal values can be taken and the competitive equilibrium is characterized by the following $n+1$ equations:

$$\sum_{i=1}^{n} x_{ij}^* + c_j^* + \bar{e}_j = x_j^*, \, \forall j \tag{2.31}$$

$$\sum_{i=1}^{n} \ell_j^* = \bar{\ell} \tag{2.32}$$

[4] A similar interpretation is valid for the production side. The ratio of expenses for factor x_{ij}^* with respect to output value $p_j x_j$ equals the exponent α_{ij}.

The optimal values depend on the wage rate and prices only, so (2.31) and 2.32) represent basically a nonlinear system of $n+1$ equations in the $n+1$ unknown prices and the wage. In a next step, the excess demand functions for all goods and labour will be formulated.

$$d_j(\mathbf{p}, w) = \sum_{i=1}^{n} x_{ij} + c_j + \bar{e}_j - x_j, \forall j \tag{2.33}$$

$$d_\ell(\mathbf{p}, w) = \sum_{i=1}^{n} \ell_j - \bar{\ell} \tag{2.34}$$

The vector function including these $n+1$ functions will be denoted by $\mathbf{d}(\mathbf{p}, w)$. Excess demand is homogeneous of degree zero, so only relative prices matter. In an equilibrium, optimal prices and the wage rate are such that the following condition is met.

$$\mathbf{d}(\mathbf{p}^*, w^*) = \mathbf{0} \tag{2.35}$$

The equilibrium price vector $\mathbf{p}^*$ in combination with the equilibrium wage w^* clear all markets. It is worth noting, that if n markets are cleared, the remaining market is automatically in equilibrium as well. Furthermore, Walras' Law holds for any set of positive prices $\mathbf{p}$ and any positive wage w, that is:

$$(\mathbf{p}, w) \cdot \mathbf{d}(\mathbf{p}, w) = 0 \tag{2.36}$$

The equilibrium of the economy at hand is given by a set of prices $\mathbf{p}^*$ and a wage w^*, such that

- the profits of all companies are maximized and problem (2.20) is solved,
- the household utility is maximized and (2.25) is solved, and
- all markets including the labour market clear.

According to Arrow & Debreu (1954), the existence of an equilibrium is assured.

Such an equilibrium can be depicted in the following table.

Table 2.1 shows the equilibrium quantities of the general equilibrium. The matrix of the x_{ij} values shows the use of intermediate goods and the row vector of the ℓ_j entries indicates the allocation of labour. The column vectors of the c_j and e_j represent the quantities of consumption goods and the exogenously given demand. The entries of each row sum up to total demand of each good that must equal supply given in the last column.

If prices are also taken into account, the equilibrium can be displayed in an even more comprehensive way as shown in table 2.2.

Table 2.1: Quantities in the Competitive General Equilibrium

x_{11}	x_{12}	x_{13}	$\dots$	$\dots$	x_{1n}	c_1	e_1	x_1
x_{21}	x_{22}				x_{2n}	c_2	e_2	x_2
x_{31}		$\dots$			$\dots$	$\dots$	$\dots$	$\dots$
$\dots$			$\dots$		$\dots$	$\dots$	$\dots$	$\dots$
$\dots$				$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
x_{n1}	$\dots$	$\dots$	$\dots$	$\dots$	x_{nn}	c_n	e_n	x_n
ℓ_1	ℓ_2	$\dots$	$\dots$	$\dots$	ℓ_n			

Table 2.2: The Competitive General Equilibrium as Input-Output Table

$x_{11}p_1$	$x_{12}p_1$	$x_{13}p_1$	$\dots$	$\dots$	$x_{1n}p_1$	c_1p_1	e_1p_1	x_1p_1
$x_{21}p_2$	$x_{22}p_2$				$x_{2n}p_2$	c_2p_2	e_2p_2	x_2p_2
$x_{31}p_3$		$\dots$			$\dots$	$\dots$	$\dots$	$\dots$
$\dots$			$\dots$		$\dots$	$\dots$	$\dots$	$\dots$
$\dots$				$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
$x_{n1}p_n$	$\dots$	$\dots$	$\dots$	$\dots$	$x_{nn}p_n$	c_np_n	e_np_n	x_np_n
$w\ell_1$	$w\ell_2$	$\dots$	$\dots$	$\dots$	$w\ell_n$			
π_1	π_2	$\dots$	$\dots$	$\dots$	π_n			
x_1p_1	x_2p_2	x_3p_3	$\dots$	$\dots$	x_np_n			

The equilibrium has the structure of a symmetric Input-Output table known in National Accounting. It shows a matrix of monetary flows of intermediate goods $x_{ij}p_i$, and in the rows below there are wages $w\ell_j$ and profits π_j. They naturally add up to the value of the gross product x_jp_j. The columns show the payments for the deliveries of good i to the various production sectors $x_{ij}p_i$, the consumption sector c_ip_i and exogenous demand e_ip_i, which add up to the value of the gross product x_jp_j as well.

The important question is now, whether or not the technological aspects of an economy can be identified from an Input-Output table via "reverse engineering". One problem concerns of course the relationship between prices and quantities. The results of any parameter identification will depend on the definition of the unit of measurement.

Nevertheless, the utilization of a Cobb-Douglas approach allows for identifying the technological parameters of the production function without choosing any unit of measurement. The optimal factor shares are related to the factor elasticities in the following way:

$$\alpha_{ij} = \frac{x_{ij}p_i}{x_jp_j}, \qquad \forall i, j \tag{2.37}$$

So under the assumption of a price taking equilibrium with profit maximization of all companies, all factor elasticities can immediately be calculated. It is worth noting, that the values of the elasticities of substitution are the sames ones than the coefficients of traditional Input-Output analysis with a Leontief technology. Nevertheless, their interpretation is different. In the case of a Leontief technology, doubling all inputs (including labour) would double the output due to the constant returns to scale. In the Cobb Douglas model under considerartion, the production function is homogenous of degree $s < 1$ and doubling the inputs would increase the output by the factor 2^s. Thus, an increase of 1% of factor i in the production of factor j would raise the output by α_{ij}%. With a Leontief technique, an increase of a single factor would not change the output because the elasticity of substitution is zero.

The similarity of the IO coefficients and the parameters of the generalized Cobb-Douglas function was already shown by Klein (1952-53) and a discussion emerged after the note of El-Hodiri & Nourzad (1988), including the comments of Kim & Hanseman (1990) and Guccione (1990).

Furthermore, the exponents of the utility function are determined similarly.

$$\beta_j = \frac{c_j p_j}{\mathbf{pc}} \tag{2.38}$$

In Leontief models, consumption is considered to be more or less exogenous. There is no theory of demand and thus no corresponding parameters can be found in those approaches. In dynamic Input-Output models it is often postulated that households consume in fixed proportions and that consumption follows a given growth rate (Dobos & Floriska 2005).

The remaining variables are the technical parameters α_j and the prices as well as the wage rate. Given prices, all quantities can easily be calculated by means of the entries in the Input-Output table. There exists no unique solution for the technical coefficients and all prices, because they are a measurement convention and may be chosen arbitrarily. The conditions for profit and utility maximization were already used for calculating the elasticities of the production and consumption functions. There are n conditions for market clearing left in order to determine either the technical coefficients or prices and the wage. The technical coefficients α_j will depend on the prices chosen in the following way.

$$\alpha_j = \left(\frac{x_{ij} p_i}{\alpha_{ij}}\right)^{1-s_j} p_j^{-1} \prod_{k=1}^{n+1} \left(\frac{p_k}{\alpha_{kj}}\right)^{\alpha_{kj}}, \forall i, j \tag{2.39}$$

$x_{ij} p_i$ is known form the Input-Output table. It is easy to see that α_j is homogenous of degree zero in prices. Furthermore, the technical scaling factor α_j depends on any price p_i.

$$\frac{\partial \alpha_j}{\partial p_i} = \alpha_{ij} \frac{\alpha_j}{p_i}, \forall \alpha_j \text{ with } i \neq j \tag{2.40}$$

$$\frac{\partial \alpha_j}{\partial p_j} = (\alpha_{jj} - 1) \frac{\alpha_j}{p_j}, \forall \alpha_j \tag{2.41}$$

It is more intuitive to consider the elasticities of α_j with respect to a change in price p_i respectively p_j.

$$\frac{\partial \alpha_j}{\partial p_i} \frac{p_i}{\alpha_j} = \alpha_{ij}, \, \forall \alpha_j \text{ with } i \neq j \tag{2.42}$$

$$\frac{\partial \alpha_j}{\partial p_j} \frac{p_j}{\alpha_j} = \alpha_{jj} - 1, \, \forall \alpha_j \tag{2.43}$$

A 1% rise of price p_i will thus increase the efficiency factor of sector j by α_{ij}% for a given table. A 1% price increase of the output of sector j will decrease its efficiency by $(1 - \alpha_{jj})$ %.

In a static framework, the price levels are not as important as they are in a dynamic framework or in international comparisons, since efficiency parameters change as soon as prices are effected. For example, when tables of two years are used, the growth rates of sectoral efficiency, expressed in the parameters α_j, can only be assessed correctly if the same unit of measurement is used, i.e. when corresponding price indices are known.

The CGE approach suffers from similar weaknesses than the structural decomposition analysis in Input-Output theory: the extraction of appropriate price indices. Efficiency increases in terms of an augmentation of the scaling factor α cannot be explained when two tables for different years are given. Nevertheless, the Cobb Douglas approach has an important advantage over traditional Input-Output analysis, since the production parameters derived do not depend on prices. Their interpretation is always the same, no matter in which country or which year they are calculated. A 1% increase of factor i will raise the output of sector j by α_{ij}%. This interpretation is invariant with respect to the units of measurement, prices and quantities produced. The comparability of IO coefficient usually causes problems as pointed out in Augustinovics (1970) or Carter (1970). Under restrictive assumptions, an analysis of structural change is nevertheless possible (Palmer & Feldman 1985).

When Leontief coefficients are extracted from Input-Output tables, they are usually value coefficients and not technical coefficients. The interpretation of the value coefficient a_{ij} is how many € input of commodity i are necessary to produce one € of output of commodity j. This interpretation obviously becomes obsolent when relative prices change. The technical coefficient t_{ij} tells one how many units of good i are necessary for one unit of output of good j. This interpretation is not appropriate when measuring units are different across countries and, furthermore, technical coefficients cannot be constructed without knowing price indices.

3 Empirical Results

This chapter presents empirical implementations of the aspects of the previous chapter 2. It is divided into two sections, the first one dealing with the interpretation of IO coefficients as factor elasticities, and the second one comparing IO and CGE models with the same data set. In both sections the economies of Austria and the USA are considered and international respectively intertemporal comparisons are made.

The information contained in Input-Output tables has been been extensively used for showing structural changes since many years. Structural decomposition analyses (SDA) haven been made in many areas (Augustinovics 1970, Carter 1970, Palmer & Feldman 1985). Besides these approaches also neoclassical total factor productivity (TFP) analyses have been utilized in order to identify structural changes in economies (ten Raa 2004).

3.1 Austria and the USA as Cobb-Douglas Economies

3.1.1 Data Sources

The study is based on the data set from the 2006 edition of OECD Input-Output tables (OECD 2006) for Austria and USA. The OECD data include symmetric Input-Output tables for the years 1995 and 2000 according to the International Standard Industrial Classification of all Economic Activities (ISIC Revision 3). More information on the data coverage is provided in the OECD Manual (Yamano et al. 2007) and an excellent introduction to Input-Output tables is provided in the Eurostat manual (Eurostat 2008). For the analyses and the models two different levels of aggregation were chosen. The analysis of factor elasticities, production and utility functions was carried out with 38 different production sectors. OECD Input-Output tables usually contain 48 production sectors, but due to differences in compilation and comparability further aggregation was necessary. The details of the aggregation can be seen in the table 5.1 on page 34. For a comparison of the linear Leontief and the nonlinear CGE model in section 3.2, a table similar to 3.1 with 11 aggregated sectors was chosen. The classifications can be found in table 5.2 on page 35.

Furthermore, adjustments of the final uses and the value added positions were necessary for handling the data. OECD tables typically differentiate between the following final use classes.

- Final consumption expenditure by households

- Final consumption expenditure by non-profit organizations serving households
- Final consumption expenditure by government
- Gross fixed capital formation
- Changes in inventories
- Valuables
- Exports
- Imports (minus)

The first two categories were aggregated to a general class *Consumption* and the remaining ones were added to autonomous or *Exogenous Final Use*. The different value added and adjustment positions in OECD tables are given by:

- Other adjustments (purchases on the domestic territory by non-residents)
- Non-comparable import (cif/fob adj direct purchases abroad by residents)
- Net taxes on products
- Compensation of employees
- Net taxes on production
- Gross operating surplus

For the purpose of the analysis in this study, the compensation of employees becomes the position *Wages* and all other categories are aggregated to *Profits*. The new aggregated tables with 38 production sectors have the following shape, given in table 3.1.

After the aggregation of various IO accounts, table 3.1 is basically identically to the representations of the CGE model in table 2.2 on page 15. The number of commodities is now $n = 38$, respectively $n = 11$ in the smaller CGE model presented on page 29. In the remaining pages of this section, 38 sector tables will be used in order to determine the production functions and the efficiency parameters of Austria and the USA. The information incorporated in production functions does not depend on prices or quantities and thus facilitates intertemporal and international comparisons.

The parameters of production functions are independent of actual prices or quantities produced, which makes them most appropriate for comparisons. They were calculated for 38 sectoral tables of Austria in the years 1995 and 2000 as well as for the USA in the same

Table 3.1: Input-Output Table used for the Analysis of Elasticities

Sector	1	2	...	38	**Intermediate Demand**	Consumption	Exogenous Final Use	**Industry Output**
1	...							
2		...						
...			...					
38				...				
	Intermediate Use							
	Wages							
	Profits							
	Industry Output							

years. This section aims at showing the main results of the intertemporal and international comparisons.

It is useful to adopt some notational conventions. The coefficient α_{ij} is defined analogously to section 2.2 as the input elasticity of good i in the production of good j. The column vector $\boldsymbol{\alpha}_{.j}$ describes the elasticities of material inputs in the production of commodity j. Furthermore we define the row vector $\boldsymbol{\alpha}_{i.}$ that contains all the elascticities of the input i in the various goods. The matrix $\boldsymbol{A}$ is defined as $(\boldsymbol{\alpha}_{.1}, \ldots, \boldsymbol{\alpha}_{.n})$. Note that $\boldsymbol{A}$ is equivalent to the coefficient matrix of Leontief Input-Output models. $\boldsymbol{\alpha}_{\ell}$ is the row vector of the factor elasticities of labour in all sectors and $\boldsymbol{\alpha}_{\pi}$ is the row vector of profit shares of all sectors. This vector can also be represented by $\boldsymbol{\alpha}_{\pi} = (1 - s_1, \ldots, 1 - s_n)$, where s_j is the degree of homogeneity in sector j. The elasticities of the utility functions for each consumption good are given in the column vector $\boldsymbol{\beta}$. When comparisons between different points of time or different countries are made, superscripts will be used to indicate the country (AUT respectively USA) and the year (95 respectively 00).

3.1.2 The Austrian Economy in 1995

With the help of the 2006 edition of the OECD Input-Output tables and the aggregation methods discussed earlier, the coefficients $\boldsymbol{A}$, $\boldsymbol{\alpha}_{\ell}$, $\boldsymbol{\alpha}_{\pi}$ and $\boldsymbol{\beta}$ were calculated for Austria for the years 1995 and 2000. The main results of this analysis will be discussed in this subsection.

In the year 1995, the production elasticities of the physical inputs in the 38 sectors ranged from nearly 0 and 0.456. The highest value is the elasticity of sector 2 (Mining and quarrying

- energy)[1] in the production of sector 8 (Coke and refined petroleum products), which is not surprising when the physics of the production technology in this sector is considered. The average factor elasticity (and also value share) of physical factors is 0.013, with a standard deviation of 0.033, showing the huge differences of factor productivities. If the rows of $\boldsymbol{A}$ are analysed, it turns out that the most productive inputs are the goods 22 (Wholesale, retail trade and repairs) as well as 29 (Finance & insurance) with average factor elasticities of 0.039 respectively 0.038. Very low average factor elasticities (0.001-0.002) can be found in the sectors 35-37 (Public administration & defence; Education; Health & social work), because these inputs are mostly part of government purchases. The shares of these goods are consequently very high in the exogenous final demand.

The vector $\boldsymbol{\alpha}_\ell$ shows the factor elasticities of labour in all 38 industries. The average value is 0.304 with a standard deviation of 0.145. Labour seems to be most productive in the sectors 35-37 with elasticities of 0.546, 0.791 and 0.486. Labour inputs have the lowest factor productivity in sector 30 (Real estate services) with a value of 0.037 and in sector 1 (Agriculture, hunting, forestry and fishing) with 0.095. The low factor productivity and value share of labour in real estate may be surprising at a first glance. It is due to the accounting of entrepreneurial income as profits and not as wages.

$\boldsymbol{\alpha}_\pi$ gives information about the profit shares. In principle, this position comprises all factors of production not taken into account as well as profits and taxes. In particular, entrepreneurial effort is captured here. The profit share is highest in sectors 30 (Real estate activities) and 31 (Renting of machinery & equipment) with values of 0.621 and 0.595. The lowest profit share was achieved in industry 2 (Mining and quarrying - energy), where even a loss incurred (-0.011). The average profit share is 0.197. The interpretation of these values is not straightforward and might also be misleading due to the aggregation of many positions (see the listing on page 19).

Value added, or household income in this model is the sum of wages and profits. The corresponding value shares are given in the sum $\boldsymbol{\alpha}_\ell + \boldsymbol{\alpha}_\pi$. The average value added is 0.501, so half of the value of the gross output is a surplus over material inputs. The highest value added (0.847) is achieved in sector 36 (Education), which is of course due to the high share of wages (0.791). The profit share in this sector is with a value of 0.056 relatively small. The lowest value added share (0.247) is earned in sector 4 (Food products, beverages and tobacco).

The degree of homogeneity of the production function of sector j is the sum of the elasticities of the inputs α_{ij}, $i = 1, \ldots, n$ plus the factor elasticity of labour $\alpha_{\ell j}$. The average value is 0.803, while the lowest value is given with 0.395 in real estate services. Increasing returns to scale seem to occur in sector 2 (Mining and quarrying - energy) with a value of 1.011.

The elasticities fo the Cobb-Douglas utility function of the household sector are given in $\boldsymbol{\beta}$. The highest elasticity can be found for the goods of sector 22 (Wholesale, retail trade

[1]Sectors are numbered according to 38 sector classification given in table 5.1 on page 34. This table also provides the corresponding OECD and ISIC Rev. 3 classifications.

and repairs) with a value of 0.197. Very low elasticities are calculated for various sectors that mainly produce intermediate goods, which are rarely demanded by consumers.

3.1.3 The Austrian Economy in 2000

By the year 2000, some changes had occurred although the main structures of the economy did not differ significantly from 1995. The average factor elasticity of physical inputs increased slightly from 0.013 to 0.014 as well as the standard deviation with a rise from 0.033 to 0.037. Again, product 2 has the highest factor elasticity in the production of sector 8 with a value of 0.481, compared to 0,456 in 1995. In general, the increase of elasticities of physical factors can be interpreted as a faster technical progress in the use of materials compared to labour or entrepreneurial inputs. In terms of the rows of the matrix of factor elasticities $\boldsymbol{A}$, no significant changes occurred. The correlation between $\boldsymbol{A}^{95}$ and $\boldsymbol{A}^{00}$ is 0.93.

The average labour productivity decreased from 0.304 in 1995 to 0.261 in 2000. The structure of the elasticities of labour remained approximately the same and the correlation between $\boldsymbol{\alpha}_\ell^{95}$ and $\boldsymbol{\alpha}_\ell^{00}$ is 0.91. Technical progress affecting labour input was probably slower than the progress influencing material inputs, so the relative value share of labour decreased. Given the high correlation, it seems that this development applies to all sectors of the economy in a similar way. One reason for this decrease may be a shift from labour in put to entrepreneurial inputs.

The average profit share increased slightly from 0.197 to 0.204. The highest values are found again in the sectors 30 and 31. The lowest profit shares are given in sectors 36 (Education) and 37 (Health & social work) with share of 0.055 and 0.063. Sector 2 that had a negative profit share in 1995 has recovered and shows a value of 0.232 in 2000. The correlation between $\boldsymbol{\alpha}_\pi^{95}$ and $\boldsymbol{\alpha}_\pi^{00}$ is 0.872 and indicates that besides the increase no substantial changes occured.

Preferences remained nearly the same over time. The correlation between $\boldsymbol{\beta}^{95}$ and $\boldsymbol{\beta}^{00}$ is 0.992, so the value share households tend to spend on various sectors can be assumed to be stable over time, at least in the medium term.

3.1.4 Structural Change in the Austrian Economy

This subsection focuses attention on the structural changes between 1995 and 2000, represented in changes of the parameters $\boldsymbol{A}$, $\boldsymbol{\alpha}_\ell$, $\boldsymbol{\alpha}_\pi$ and $\boldsymbol{\beta}$.

In a first step, the growth rates of the rows of $\boldsymbol{A}$ will be examined. This analysis shows which factor elasticities have grown the most during the period of consideration. For any matrix $\boldsymbol{M} = \{m_{ij}\}$ we define $\hat{\boldsymbol{M}}$ to be the matrix of annual growth rates of its elements.

$$\hat{\boldsymbol{M}} = \{\hat{m}_{ij}\}, \quad \text{with} \quad \hat{m}_{ij} = \left(\frac{m_{ij}^{00}}{m_{ij}^{95}}\right)^{\frac{1}{5}} - 1 \tag{3.1}$$

For the Austrian case, the matrix $\hat{\boldsymbol{A}}$ was calculated in order to examine the technical developments. The productivity of commodity 15 (Office, accounting & computing machinery) increased in 34 of the 38 sectors with an average growth rate of 13% per year. The factor elasticity of sector 17 (Other business activities) even increased in 35 sectors, though the average growth rate was 6.5%. In comparison to other inputs, the good of sector 29 (Finance & insurance) had its productivity decreased in 36 sectors with an average decline of -2.5% per year. Sector 35 (Public administration and defence; compulsory social security) had an average decline of 9.2%.

This approach of examining columns and rows enables one to differentiate between technical progress related to the production function of an industry (column) and developments that are specific to a special input factor (row). Technical improvements in an industry in general will be characterized by an increased value added share, while improvements of a factor will raise its elasticity in most industries.

In terms of changes in the vector of factor elasticities of labour $\boldsymbol{\alpha}_\ell$, the highest productivity took place in sector 30 (Real estate activities), where the average growth rate was 6.5% per year. Though, as already discussed previously, the labour productivity started from a very low level in that industry. Annual decreases between 13% and 15% are identified in the sectors 15 (Office, accounting & computing machinery), 25 (Water transport) and 28 (Post & telecommunications).

The growth rates of $\boldsymbol{\alpha}_\pi$ show the evolution of the profit shares of all sectors. The industries 4 (Food products, beverages and tobacco) and 5 (Textiles, textile products, leather and footwear) experienced the largest annual increases with growth rates of 11.3% and 10.6%. A more detailed discussion is difficult because the definition of profits is very broad and it also comprises several adjustment positions (see the listing on page 19).

The change in consumer preferences can be seen in the evolution of $\boldsymbol{\beta}$. The highest average annual growth rate is observed in the elasticity for goods of sector 15 (Office, accounting & computing machinery), indicating a growing demand for personal computing. The largest decrease concerned public administration and defence (sector 35) with an annual decrease of 13.9% of the corresponding elasticity.

3.1.5 The Economy of the USA in 1995

A similar analysis as in the previous subsection was also carried out for the USA. First, the year 1995 will be analysed, then the year 2000 and the structural changes will be discussed in greater detail.

The matrix $\boldsymbol{A}$ for the USA in the year 1995 reveals that the factor elasticities of physical inputs ranged between values close to zero and a maximum of 0.520. Not very surprisingly, this coefficient is α_{28}, which was also the highest value in Austria in 1995. The average factor elasticity is 0.013 with a standard deviation of 0.033, just like in Austria. The sector with the highest factor productivity on average is 22 (Wholesale, retail trade and repairs) with an average factor elasticity of 0.049. Low elasticities can be found in many service related industries.

The information contained in $\boldsymbol{\alpha}_\ell$ shows an average elasticity of labour of 0.29 (compared to 0.304 in Austria this time). The highest wage share can be found in education (sector 36) and the lowest one (0.04) in real estate services. In terms of profit shares, the highest one (0.766) is given in real estate and the low profit shares (0.033-0.038) characterize the industries 16 (Electrical machinery) and 32 (Computer & related activities). The average profit share is 0.198 and correspondingly, the average homogenity of the production functions is 0.802 (compared to 0.803 in Austria).

The vector $\boldsymbol{\beta}$ indicates that preference parameters are highest for sector 22 (Wholesale, retail trade and repairs), 37 (Health & social work) and 30 (Real estate activities) with values of 0.176, 0.165 and 0.143. Low values close to zero can be found in R&D (sector 33) and public administration (sector 35).

3.1.6 The Economy of the USA in 2000

Similar to Austria, few radical changes happened to the production structure, and the correlation between $\boldsymbol{A}^{95}$ and $\boldsymbol{A}^{00}$ is 0.87. The average factor elasticity of physical inputs remains 0.013 with a standard deviation of 0.033. The maximum coefficient α_{28} increased to 0.619.

The average wage share has slightly fallen from 0.29 to 0.288 and the correlation between $\boldsymbol{\alpha}_\ell^{95}$ and $\boldsymbol{\alpha}_\ell^{00}$ is 0.869. Given the nearly constant wage share, the technical progress of labour inputs seems to be as high as the progress affecting physical inputs.

The average profit share increased to 0.2 compared to 0.198 in the year 1995. The highest profit share can still be found in real estate services and the correlation between $\boldsymbol{\alpha}_\pi^{95}$ and $\boldsymbol{\alpha}_\pi^{00}$ is 0.948, indicating that no major global changes occurred.

In terms of consumption preferences, there is a correlation of 0.98 between the parameters of the year 1995 and 2000. Goods of sector 22 (Wholesale, retail trade and repairs) still have the highest elasticity with a value of 0.161.

3.1.7 Structural Change in the Economy of the USA

Similar to Austria, it will be analysed which factor elasticities increased or decreased most on average in order to show productivity gains. The information on these developments can be obtained from the matrix $\hat{\boldsymbol{A}}^{USA}$. Interestingly, the annual growth rates of the factor elasticity of sector 35 (Public administration and defence and compulsory social security) increased in all 38 industries with an average growth rate of over 400,000%(!), which can probably be accounted to changes in the compilation of national accounts rather than to technical progress. The factor productivity of sector 33 (Research & development) also rose in all industries with an average growth rate of 110.2%. Finally, similar to Austria, the factor elasticity of office, accounting and computing machinery (sector 15) increased in 34 sectors with an annual growth rate of 45.3% (compared to 13% in Austria).

Changes in the labour productivity were largest in sector 15 with an annual increase of 13% and lowest in industry 16 (Electrical machinery) with a decrease of 11.1% per year. Furthermore, declines are found in sectors 25 (Water transport) and 28 (Post & telecommunications). Profit shares increased most in industry 16 and in 33 (Research & development).

The change of $\boldsymbol{\beta}$ shows that consumers' preferences for R&D and public administration increased. Though, this change might also be due to a different compilation of these goods in national accounting. Similar to Austria, preferences also shifted to office, accounting and computing machinery (sector 15) with an increase of 36.5% per annum of the elasticity. A decline of preferences is visible for sector 21 (Construction).

3.1.8 Convergence or divergence between Austria and the USA?

In this subsection, similarities and differences of the Austrian and USA economy will be shown. First, the factor parameters of each year will be compared and in a second step, the growth rates will be analyzed.

In the year 1995, there seems to be many similarities of the production structures. The correlation of the technological matrices $\boldsymbol{A}^{AUT}$ and $\boldsymbol{A}^{USA}$ is 0.831, indicating that there is a clear linear realtionship. If the column vectors $\boldsymbol{\alpha}_{.j}$ are taken into account (all factor elasticities in the production of good j), their average is 0.913, with 21 industries that have a correlation of 0.95 or higher. In terms of the row vectors $\boldsymbol{\alpha}_{i.}$ (the elasticities of good i in all sectors), the average correlation is 0.716 and 8 goods have a correlation of 0.95 or above.

The correlation between the wage shares is 0.726 and profit shares are linearly related with a correlation coefficient of 0.684. In 1995, the correlation between the consumption parameters $\boldsymbol{\beta}^{AUT}$ and $\boldsymbol{\beta}^{USA}$ amounts to 0.854 and even the value shares of exogenous consumptions correlate well (0.927), indicating that expenditure preferences are similar across countries.

In the year 2000, the correlation between the technological matrices has decreased to 0.761. The column vectors $\boldsymbol{\alpha}_{.j}$, $j = 1, \dots n$, still have a high correlation of 0.881 and 20 industries have correlations of 0.95 or above. Some industries still use very similar technologies, while a few developed differently. For example $\boldsymbol{\alpha}_{.26}^{AUT}$ and $\boldsymbol{\alpha}_{.26}^{USA}$, the vectors of elasticities of industry 26 (Air transport) even have a negative correlation (-0.272) in 2000. In 1995 it was 0.604, so drastic changes must have occurred in that sector.

The row vectors $\boldsymbol{\alpha}_{i.}$ also have a lower average correlation of 0.632, although the number of goods with correlations of 0.95 or higher increased from 8 to 12. This indicates that the use of some products might have become more similar both economies. For example, the factor elasticities of good 5 (Textiles) or 17 (Motor vehicels and trailers) in the production of the other goods correlate with a coefficient of 0.996 across borders.

In terms of wage shares and profit shares, the correlation of labour elasticities has fallem from 0.726 to 0.681, while profit shares still show a moderate positive correlation of 0.684 (compared 0.695 in 1995). The correlation between the parameters of the utility function even increased slightly from 0.854 to 0.856 and household preference seemed to have developed in a similar direction.

After this comparison of static values, the development of the technological parameters of both economies will be examined. The interesting question at hand is where there is convergence in technical developments and where divergence can be found. Furthermore, it can be examined if rather industries (columns) or products (rows) develop into similar directions. For this purpose, the matrices of growth rates $\hat{\boldsymbol{A}}^{AUT}$ and $\hat{\boldsymbol{A}}^{USA}$ are compared. Similarities in columns show that industries follow the same trend while analogies in rows indicate common practices of a certain good.

A first statement can be made about the correlation between $\hat{\boldsymbol{A}}^{AUT}$ and $\hat{\boldsymbol{A}}^{USA}$. It is basically zero with a value of 0.037, so there is no systematic linear relationship between the growth rates of factor elasticities. The same is true, if the correlation coefficient between the column vectors $\hat{\boldsymbol{\alpha}}_{.j}$ of both economies is considered (-0.097) or if the row vectors $\hat{\boldsymbol{\alpha}}_{i.}$ are taken into account (0.024). From this aggregate point of view, there seems to be no convergence at all and the technical developments are more or less purely random.

Between the row vectors $\hat{\boldsymbol{\alpha}}_{.j}$, weak positive correlations (0.36 and 0.26) can be found in the sectors 35 (Public administration) and 29 (Finance & insurance). Especially in sector 29 this result can be explained by the international character of the finance business. The row vectors $\hat{\boldsymbol{\alpha}}_{i.}$ have moderate correlations for the use of good 33 (R&D) and 35 with coefficients of 0.266 respectively 0.616. Furthermore, there is no significant linear relationship between the development of the technical parameters. The same is also true for the growth of wage and profit shares and the development of consumer preferences.

3.2 A Comparison of the IO and the CGE Approach

In this section, we discuss the differences between Input-Output models and the computable general equilibrium approach used in this study. The main points are summarized in table 3.2.

The data for both approaches are collected from OECD Input-Output tables of the year 2000 (Yamano et al. 2007, p.27&52). The dataset is available in tables 5.3 and 5.4. From these tables we calculated the value Input-Output coefficients and the factor elasticities that are given in the matrices $\boldsymbol{A}^{AUT}$ and $\boldsymbol{A}^{USA}$ that can be found on page 37. Furthermore, we identified labour coefficients respectively factor elasticities $\boldsymbol{\alpha}_{\ell}$. These parameters are needed for both approaches, though, in different way. The exogenous expenses (in value terms) $\bar{e}_j p_j$, $\forall j$ are also used in both models. Differences in the building blocks concern the consumption. In the Input-Output model, consumption is given as an exogenous vector $\mathbf{c}$, while in the CGE

Table 3.2: Differences between IO and CGE Models

	Input-Output Analysis	**Computable General Equilibrium**
Exogenous Data	Input-Output coefficients ($n \times n$) Labour coefficients (n) Consumption quantities (n) Exogenous demand (n) Prices and the wage rate($n+1$)	Factor elasticities of physical inputs ($n \times n$) Factor elasticities of labour (n) Elasticities of utility function (n) Exogenous demand (n) Scaling parameters (n) Labour supply (1)
Endogenous Data	Output quantities (n) Intermediate demand ($n \times n$) Wages (n) Profits (n) Labour demand (1)	Output quantities (n) Intermediate demand ($n \times n$) Wages (n) Profits (n) Prices and the wage rate ($n+1$) Consumption quantities (n)
Functional Forms	Linear	Non-Linear
Response	Quantity changes	Quantity and price changes

model, the parameters β are calculated from these values.

Similar to the previous section, we use these parameters to compare the structure of both economies. Not suprisingly, due to the higher level of aggregation, the correlation between $\boldsymbol{A}^{AUT}$ and $\boldsymbol{A}^{USA}$ has with 0.82 a very high value, indicating similar structures. This interpretation can now be done in terms of IO coefficients or factor elasticities.

The Leontief inverse matrices are even more closely related and show a correlation coefficient of 0.991. The Leontief matrices are given in chapter 5. The interpretation of the Leontief inverse is of course limited to the Input-Output model and there is no corresponding set of parameters in the CGE model.

3.2.1 Results of the IO Model

Traditional Input-Output models describe economies in terms of systems of linear equations. Given a matrix $\boldsymbol{A}$ of production coefficients, the following matrix equation is assumed to hold.

$$\boldsymbol{A}\boldsymbol{x} + \boldsymbol{c} = \boldsymbol{x} \tag{3.2}$$

Equation (3.2) states that the total output $\boldsymbol{x}$ is composed of intermediate products $\boldsymbol{A}\boldsymbol{x}$ plus final demand $\boldsymbol{c}$. The matrix $\boldsymbol{A}$ is assumed to be constant and represents the production technology of the economy. (3.2) can be rearranged in order to show how simulations can be made in an IO model.

$$\boldsymbol{x} = (\boldsymbol{I} - \boldsymbol{A})^{-1}\,\boldsymbol{c} \tag{3.3}$$

Total output depends on the final demand of the economy. Any exogenous change affects production quantities and usually imposes severe changes on the economy. If exogenous consumption expenditures $\bar{\boldsymbol{e}}$ are added to the original consumption levels, the following output changes $d\boldsymbol{x}$ occur.

$$d\boldsymbol{x} = (\boldsymbol{I} - \boldsymbol{A})^{-1}\,\bar{\boldsymbol{e}} \tag{3.4}$$

The so-called *Leontief Inverse* $(\boldsymbol{I} - \boldsymbol{A})^{-1}$ describes this linear relationship. The IO model is especially useful for evaluating the impact of increased government spendings, but it is limited by the assumption of non-substitution between factor inputs.

In the report at hand, the technological matrices $\boldsymbol{A}$ were calculated for Austria and the USA in the year 2000. They are given in tables 5.5 and 5.6 on page 37. They are calculated from the monetary Input-Output tables, so they are value coefficients. α_{ij} indicates how many € (respectively \$) value of product i are needed to produce one € (respectively \$) value of product j. The coefficient matrices are followed by the corresponding Leontief Inverse matrices.

Given these coefficient matrices, it is easy to determine changes corresponding to increasing exogenous consumption. In the present study, the consequences of changes in the consumption pattern were calculated for all 11 sectors. The new exogenous demand for each single sector is equal to 10% of total output in this sector. Furthermore, the effect of a simultaneous increase in all sectors corresponding to 1% was calculated. The results for the Austrian economy in the year 2000 is given in table 3.3.

Table 3.3: Output Changes in the IO Model for Austria in 2000

	10% increase of sector											**1% increase**
Sector	1	2	3	4	5	6	7	8	9	10	11	**of all sectors**
1	12.8%	2.4%	1.4%	0.5%	2.3%	0.6%	0.4%	0.3%	0.2%	0.2%	0.3%	3.9%
2	1.0%	13.0%	0.6%	0.4%	0.3%	0.7%	0.6%	0.3%	0.2%	0.2%	0.3%	1.9%
3	1.9%	2.6%	15.5%	3.9%	1.1%	3.4%	1.5%	1.6%	0.8%	1.1%	1.1%	2.8%
4	0.8%	0.7%	0.8%	15.0%	0.8%	1.0%	0.8%	1.1%	0.3%	0.4%	0.5%	2.1%
5	0.4%	0.4%	0.5%	0.3%	14.3%	0.2%	0.3%	0.3%	0.2%	0.2%	0.4%	2.6%
6	0.3%	0.3%	0.3%	0.2%	0.3%	10.9%	0.3%	0.4%	1.0%	0.2%	0.4%	1.5%
7	1.0%	1.3%	1.0%	1.3%	0.7%	0.9%	11.0%	1.4%	0.3%	0.5%	0.5%	1.6%
8	0.7%	0.9%	1.1%	0.8%	0.6%	0.6%	0.8%	12.8%	0.3%	0.5%	0.4%	2.1%
9	0.9%	1.1%	1.1%	1.1%	0.9%	1.0%	1.6%	1.2%	12.0%	1.3%	0.9%	2.1%
10	0.7%	1.1%	1.2%	1.3%	0.7%	1.0%	1.2%	1.3%	0.7%	12.5%	0.8%	2.7%
11	0.3%	0.3%	0.4%	0.3%	0.2%	0.2%	0.3%	0.2%	0.4%	0.4%	10.7%	1.2%

The corresponding values for the USA in the year 2000 are given in table 3.4.

The interpretation of both tables is similar. Rising expenditures in one sector usually imply increases in all other sectors via inter-industry linkages. The qualitative effects do not differ significantly between Austria and the USA. There is a nearly perfect positive correlation between these results. The correlation coefficient between these matrices of results amounts to 0.99 and shows, that rises of exogenous demand have more or less the same results in both countries. This result confirms the assumption that industry structures are very similar, at least given the 11 sector classification used in this paper.

Table 3.4: Output Changes in the IO Model for the USA in 2000

	10% increase of sector											1% increase
Sector	1	2	3	4	5	6	7	8	9	10	11	**of all sectors**
1	12.6%	2.6%	1.8%	0.5%	2.6%	0.6%	0.3%	0.3%	0.2%	0.2%	0.3%	3.6%
2	0.6%	12.2%	0.4%	0.3%	0.2%	0.7%	0.5%	0.2%	0.2%	0.1%	0.3%	1.8%
3	1.9%	2.6%	14.4%	3.3%	1.1%	2.8%	1.0%	1.5%	0.6%	1.0%	1.2%	2.8%
4	0.6%	0.6%	0.7%	14.1%	0.4%	1.1%	0.6%	0.8%	0.2%	0.4%	0.6%	2.0%
5	0.3%	0.3%	0.3%	0.2%	10.2%	0.1%	0.2%	0.1%	0.1%	0.1%	0.2%	2.0%
6	0.1%	0.1%	0.1%	0.1%	0.2%	10.0%	0.1%	0.1%	0.1%	0.1%	0.2%	1.2%
7	0.9%	1.5%	1.3%	1.6%	0.5%	1.6%	10.7%	0.8%	0.4%	0.6%	0.7%	1.6%
8	0.7%	1.0%	1.0%	0.8%	1.1%	0.7%	0.6%	12.1%	0.4%	0.7%	0.7%	2.2%
9	1.7%	1.2%	1.1%	1.0%	0.9%	0.8%	1.1%	0.9%	12.5%	1.1%	1.1%	1.8%
10	1.3%	2.0%	1.8%	2.1%	0.9%	1.9%	1.5%	1.5%	1.2%	11.9%	1.5%	2.5%
11	0.3%	0.3%	0.3%	0.3%	0.2%	0.3%	0.3%	0.4%	0.2%	0.3%	10.6%	1.2%

3.2.2 The Computable General Equilibrium Model

According to the data requirements given in table 3.2, each economy is described by the following characteristics.

- A matrix of factor elasticities $\boldsymbol{A}$
- A vector of labour elasticities $\boldsymbol{\alpha}_\ell$
- Efficiency parameters α_j for each sector $j = 1, \ldots, 11$
- A vector $\boldsymbol{\beta}$ of elasticities of the utility function
- A vector $\boldsymbol{e}$ of exogenous demand
- Labour supply ℓ

From the 11 sector Input-Output table we can calculate $\boldsymbol{A}$, $\boldsymbol{\alpha}_\ell$, $\boldsymbol{\beta}$ and $\boldsymbol{e}$, but we still need the scaling factors and the labour supply in order to calculate the equilibrium. In principle, these variables could be set arbitrarily, because they merely express the conventions of measurement (eg. kilograms, pounds, hours, minutes, etc.) and do not alter the structure of the economy. In section 2.2 we already showed the relationships between these scaling parameters and prices. For our model, we will choose a more intuitive way of defining measurement units and set prices and the wage rate. The scaling factors will adjust to fit the data.[2] However, a comparison of these variables is not possible unless the same standard of measurement is chosen.

In contrast to the IO model, a crucial assumption of the CGE model is full employment. Households always supply ℓ units of labour and the wage rate will adjust, such that the market clears. In the IO model, labour supply is completely elastic and households will supply any amount of labour.

For computational reasons, all prices including the wage rate were set equal to 10. This dimension proved to facilitate the numerical solution of the problem. Given this definition,

[2] A equivalent approach is for example to set all scaling factors and to observe the corresponding prices.

we calculated scaling factors and labour supply for the economies of Austria and the USA, which completes the data set needed.

Due to the limited size of the model, we were able to calculate the solution to this model with the help of the Solver tool in Microsoft Office Excel. This was done by calculating supply and demand (intermediate plus final demand) and setting excess demand equal to zero while having the prices as choice variables. We chose the wage rate as the numéraire of the system.

For the given scaling factors, the unique solution of the equilbrium yields of course prices of 10 for every good and the corresponding quantities. For a scenario analysis, the exogenous expenditures may be varied in the analysis. This will be done in the same way as in the Input-Output analysis. The exogenous demand in each sector will be raised by an amount of 10% of the initial total output. Furthermore, calculations will also be run for a 1% increase in all 11 sectors. The results for Austria and the USA in the year 2000 are given in tables 3.5 and 3.6.

Table 3.5: Output Changes in the CGE Model for Austria in 2000

	10% increase of sector											**+1%**
Sector	1	2	3	4	5	6	7	8	9	10	11	**all**
1	5.11%	1.77%	0.76%	-2.40%	0.57%	-0.37%	-1.43%	-0.94%	-1.37%	-1.03%	-2.00%	0.02%
2	-1.05%	8.95%	-2.78%	-4.28%	-0.73%	-1.45%	-2.49%	-1.67%	-2.13%	-1.72%	-3.50%	-1.04%
3	-0.41%	-0.03%	10.58%	-0.38%	-0.33%	0.83%	-0.67%	-0.39%	-0.83%	-0.65%	-1.32%	0.77%
4	-0.13%	-0.47%	-1.12%	10.81%	-0.14%	-0.22%	-0.86%	-0.18%	-0.98%	-0.80%	-1.50%	0.57%
5	-0.85%	-0.65%	-0.68%	-2.49%	9.80%	-0.92%	-1.20%	-0.67%	-0.85%	-0.86%	-1.28%	0.09%
6	-0.11%	-0.26%	-0.65%	-1.10%	-0.09%	9.57%	-0.46%	-0.17%	0.84%	-0.36%	-0.38%	0.74%
7	-0.31%	-0.95%	-1.99%	-3.43%	-0.45%	-1.28%	5.78%	-1.05%	-2.06%	-1.46%	-3.13%	-0.82%
8	-0.23%	-0.66%	-1.13%	-3.26%	-0.35%	-1.12%	-1.75%	8.72%	-1.66%	-1.26%	-2.70%	-0.34%
9	-0.17%	-0.57%	-1.14%	-2.17%	-0.27%	-0.96%	-0.91%	-0.62%	4.12%	-0.68%	-1.71%	-0.38%
10	-0.05%	0.03%	-0.02%	-1.24%	-0.12%	-0.08%	0.15%	0.12%	-0.54%	7.92%	-0.89%	0.64%
11	-0.14%	-0.55%	-0.93%	-1.92%	-0.24%	-0.75%	-1.18%	-0.65%	-0.65%	-0.55%	8.47%	0.21%

Table 3.6: Output Changes in the CGE Model for the USA in 2000

	10% increase of sector											**+1%**
Sector	1	2	3	4	5	6	7	8	9	10	11	**all**
1	7.12%	2.09%	2.27%	-1.77%	0.82%	-0.10%	-1.09%	-0.62%	-1.96%	-1.41%	-1.68%	0.48%
2	-1.17%	8.65%	-2.41%	-3.66%	-0.48%	-0.61%	-2.02%	-1.41%	-2.42%	-2.59%	-3.61%	-0.96%
3	-0.53%	0.12%	9.72%	-0.46%	-0.24%	0.37%	-1.03%	-0.34%	-1.19%	-1.08%	-1.09%	0.55%
4	-0.18%	-0.30%	-0.77%	10.81%	-0.12%	-0.06%	-0.92%	-0.28%	-1.15%	-1.08%	-1.26%	0.58%
5	-0.89%	-0.40%	-0.61%	-1.54%	5.42%	-0.40%	-0.76%	-0.66%	-0.80%	-0.96%	-0.98%	-0.17%
6	-0.03%	-0.06%	-0.15%	-0.34%	0.03%	9.74%	-0.14%	-0.06%	0.19%	-0.13%	0.23%	0.95%
7	-0.30%	-0.45%	-1.05%	-2.52%	-0.23%	-0.44%	5.86%	-0.85%	-1.95%	-1.85%	-2.83%	-0.48%
8	-0.16%	-0.13%	-0.46%	-2.04%	0.01%	-0.33%	-0.98%	8.01%	-1.13%	-0.95%	-1.43%	0.18%
9	-0.12%	-0.35%	-0.80%	-1.80%	-0.14%	-0.45%	-1.00%	-0.54%	4.44%	-1.09%	-1.57%	-0.24%
10	-0.06%	0.09%	-0.08%	-1.49%	-0.08%	-0.02%	-0.30%	-0.24%	-0.30%	8.32%	-0.74%	0.65%
11	-0.30%	-0.61%	-1.33%	-2.68%	-0.21%	-0.70%	-1.75%	-0.84%	-1.67%	-1.65%	6.75%	-0.34%

In contrast to the IO model, these quantity changes are combined with corresponding price changes. New relative prices completely modify the optimization problems of households and companies and do not always lead to the consequences one might expected. Tables 3.5 and 3.6 clearly indicate that additional exogenous demand in one sector typically decreases the equilibrium quantities of other sectors. Even if the new demand affects all 11 sectors, the quantities of some sectors decrease as the last columns show.

Interestingly, the results of both countries do not differ a lot. Given the matrices of output changes, there is a significant positive correlation with a coefficient of 0.97. This value is only slightly lower than the corresponding coefficient of the linear IO model, so exogenous changes in final demand seem to have similar consequences. Demand focussed economic policies will have the same effects in both countries in this model.

The results of the CGE analysis exhibit in general lower impacts of changes in exogenous demand. While in the IO model an increase of exogenous demand typically raises total output of all sectors, the CGE model typically implies decreases in other sectors as a response. Nevertheless, in qualitative terms, the results of the IO analysis and the CGE are similar. The correlation coefficient of the Austrian tables 3.3 and 3.5 is 0.93. The same is true for the USA tables 3.4 and 3.6.

The IO approach probably overestimates the effects of a demand stimulus, while the CGE may underestimate its consequences. Though, the high level of correlation shows that changes affect the economy in a similar way.

4 Conclusion

The results of the research project at hand are numerous. They can be grouped into theoretical and empirical results.

First, the inability of many neoclassical models to deal with production of commodities by means of commodities was revealed. Constant returns to scale are not possible in static CES production economies that allow for substitutability of inputs. Usually, more complicated approaches involving different layers of production and the use of capital and labour as factors are implemented in order to deal with this problem. Another solution is the Leontief approach that is characterized by non-substitutability of inputs.

Second, a new interpretation of IO tables as equilibria in Cobb-Douglas economies was presented. Symmetric IO tables compiled by statistical offices are not limited to an interpretation as Leontief economies, they can also be seen as a general equilibria in a Cobb-Douglas economy with decreasing returns to scale. This approach gives rise to a more comprehensive formulation of traditional Input-Output coefficients. In the neoclassical framework, they are factor elasticities that have an important advantage over Input-Output coefficients: they are dimensionless. In this framework, international as well as intertemporal comparisons are easily possible.

Given these theoretical findings, a powerful empirical tool is introduced. The theoretical background enables a comparison of Austria and the USA for the years 1995 and 2000. This approach combines an intertemporal assessment of structural change in each single country as well as an international analysis of differences and similarities of these countries.

In the case of Austria, with the exception of few sectors, the average change of the factor elasticities is low. The productivity of sector 15 (Office, accounting and computing machinery) increased by 13% each year. Furthermore, the wage share decreased from 0.30 to 0.26 in five years and the profit share remained more or less stable at a level of 0.2. Labour is successively being replaced by physical inputs. There is no significant change in the consumption behaviour and consumer preferences expressed in the utility elasticities of all goods.

The situation of the USA is similar to the Austrian one. The productivity of computing machines also increased steadily, with an even higher growth rate of 43% per year. Additionally, the productivity of R&D as an input in production processes rose on average by 110% in contrast to Austria, where the average growth rate is 6%. Wage shares as well as profit shares remained constant at levels of 0.29 and 0.20. Consumer preferences were stable with a correlation coefficient of 0.98 of the elasticities.

In an international comparison, we find out that convergence as well as divergence between these economies takes place. Some industries become more and more similar in their production processes, while others tend to diverge. About 20 sectors have factor elasticities with correlations coefficients of 0.95 or higher. Other industries like air transport developed completely differently and seem to use totally different inputs. On an aggregated level, no convergence can be found in the technological changes in both countries. Factor productivity increases are not uniform on a global level. While some factors become more productive in Austria, others get more productive in the USA. The same statement can be made for wage and profit shares and consumer preferences. Technological improvements are either limited to geographical areas or do not affect all countries in the same way.

The second empirical implementation of the theoretical framework is the comparison of IO and CGE models. Although both approaches show different quantitative results, some qualitative aspects are very similar. Increases of exogenous demand imply much stronger quantity changes in the IO model than in the CGE approach, partly due to the price adjustments in the CGE model. From a qualitative point of view, the quantity changes show similar properties. In the linear IO model, both countries react very similarly in response to increased exogenous demand. Given the similarity of technical coefficients, one could have expected this result. Interestingly, the CGE model shows very similar results for Austria and the USA. An exogenous shock in one specific sector tends to influence all sectors, no matter in which country, in the same way. Given the nonlinear structure of the CGE model, this result is no matter of course.

The results of this research project rely on specific assumptions and simplifications that may we weakened in order to give a more realistic account of industrial dynamics. Final demand was split into consumption and exogenous demand that combined the categories investments, fixed capital formation, government expenditures and exports. Value added was defined as wages and profit that included taxes and imports. In a next step it may be fruitful to include theses aspects in the analysis. Furthermore, a Cobb-Douglas approach was chosen that is a very special case of the CES function. A more advanced model may incorporate CES production functions with different elasticities of substitution between sectors. Finally, the CGE was limited to 11 sectors and may be enlarged to incorporate all 38 sectors. For this project the use of a optimization software package will be necessary.

5 Tables

Table 5.1: Aggregation of the OECD Tables

#	Description	OECD Sector	ISIC Rev. 3 code
1	Agriculture, hunting, forestry and fishing	1	1+2+5
2	Mining and quarrying (energy)	2	10-12
3	Mining and quarrying (non-energy)	3	13+14
4	Food products, beverages and tobacco	4	15+16
5	Textiles, textile products, leather and footwear	5	17-19
6	Wood and products of wood and cork	6	20
7	Pulp, paper, paper products, printing and publishing	7	21+22
8	Coke, refined petroleum products and nuclear fuel	8	23
9	Chemicals and pharmaceuticals	9-10	24
10	Rubber and plastics products	11	25
11	Other non-metallic mineral products	12	26
12	Iron, steel and non-ferrou metals	13-14	27
13	Fabricated metal products, except machinery and equipment	15	28
14	Machinery and equipment	16	29
15	Office, accounting and computing machinery	17	30
16	Electrical machinery, radio, televison equipment and medical instruments	18-20	31-33
17	Motor vehicles, trailers and semi-trailers	21	34
18	Building & repairing of ships and boats , aircrafts and railroad and transport equipment	22-24	35
19	Manufacturing nec; recycling (include Furniture)	25	36+37
20	Production, collection and distribution of electricity, gas, water	26-29	40+41
21	Construction	30	45
22	Wholesale and retail trade; repairs	31	50-52
23	Hotels and restaurants	32	55
24	Land transport; transport via pipelines	33	60
25	Water transport	34	61
26	Air transport	35	62
27	Supporting & auxiliary transport activities; activities of travel agencies	36	63
28	Post and telecommunications	37	64
29	Finance and insurance	38	65-67
30	Real estate activities	39	70
31	Renting of machinery and equipment	40	71
32	Computer and related activities	41	72
33	Research and development	42	73
34	Other Business Activities	43	74
35	Public administration and defence; compulsory social security	44	75
36	Education	45	80
37	Health and social work	46	85
38	Other community, social and personal services; private households	47-48	90-99

Table 5.2: Input-Output Table used for the CGE Analysis

#	**Sector**	**ISIC Rev. 3 code**
1	Primary	1-4
2	Other manufacturing	15-20, 6-37
3	Material manufacturing	21-28
4	Machinery manufacturing	29-35
5	Utility	40-41
6	Construction	45
7	Trade, hotels and restaurants	50-55
8	Transport and communications	60-64
9	Finance, insurance and real estates	65-70
10	Business services	71-74
11	Personal services, Households	80-99

Table 5.3: 11 Sector Input-Output Table of Austria

Sector	#	1	2	3	4	5	6	7	8	9	10	11	**Total**	Cons..	Exog.	**Output**
Primary	1	1758	3552	2841	18	1170	463	429	50	49	8	193	10531	2138	-3541	9129
Other manufacturing	2	475	5715	1021	351	31	895	2159	187	137	99	954	12023	14481	459	26964
Material manufacturing	3	631	2482	14494	5298	190	4977	3208	1433	702	1096	2538	37049	5177	3052	45277
Machinery, manufacturing	4	268	426	921	10673	268	1335	1908	1368	220	343	1188	18918	4961	9875	33754
Utility	5	141	366	841	222	2910	92	658	264	436	113	1024	7067	2424	393	9884
Construction	6	104	205	241	124	82	2078	499	447	3256	70	1345	8451	1128	19018	28597
Trade, hotels, restaurants	7	481	1750	1920	2014	251	1334	3531	2355	369	515	1456	15976	31053	9301	56330
Transport, communications	8	223	884	2109	688	197	484	2334	5704	445	526	1118	14711	10348	3298	28356
Finance, insurance	9	321	928	1572	1069	299	1174	5350	1447	6508	1808	2919	23393	19302	272	42967
Business services	10	168	1080	1943	1500	187	1151	3599	1683	1486	4559	2373	19729	1034	3756	24518
Personal services	11	120	258	878	172	36	92	747	231	1054	608	3095	7292	12143	37124	56559
Total		4689	17646	28782	22127	5621	14076	24420	15167	14662	9744	18204	175138	104191	83006	362335
Wages		885	5828	9598	7561	2149	8298	18441	7688	7003	7455	31815	106720			
Profits		3554	3489	6898	4067	2114	6222	13470	5501	21303	7320	6540	80478			
Output		9128	26964	45277	33754	9884	28597	56330	28356	42968	24519	56559	362335			

Price Valuation: Basic prices
Currency: Millions of €

Table 5.4: 11 Sector Input-Output Table of the United States of America

Sector	#	1	2	3	4	5	6	7	8	9	10	11	**Total**	Cons.	Exog.	**Output**
Primary	1	86.4	136.5	168.7	1.3	65.6	6.2	11.8	4.9	3.2	4.2	13.6	502.4	46.3	-71.3	477.5
Other manufacturing	2	14.8	153.2	21.6	17.0	0.8	38.4	90.0	3.8	23.3	7.9	49.3	420.0	640.8	-122.1	938.7
Material manufacturing	3	38.3	105.6	481.9	235.9	10.2	129.7	101.5	72.2	57.3	85.0	192.3	1509.9	342.2	-61.2	1790.9
Machinery, manufacturing	4	9.7	14.0	42.2	443.2	4.1	48.0	71.1	43.4	15.7	36.0	95.2	822.7	315.5	478.7	1616.9
Utility	5	6.7	11.6	30.7	8.5	2.2	2.6	27.3	5.7	24.5	11.5	46.7	178.0	135.3	15.3	328.6
Construction	6	0.9	1.8	3.7	2.5	4.1	0.9	9.4	3.7	26.1	8.1	44.9	106.1	0.0	750.7	856.8
Trade, hotels, restaurants	7	19.2	75.4	117.6	131.1	4.5	91.0	104.3	50.1	47.6	66.7	117.2	824.6	1551.4	188.8	2564.7
Transport, communications	8	12.6	35.1	71.5	39.7	22.0	23.7	83.0	173.8	47.0	84.2	125.5	718.2	321.6	101.4	1141.2
Finance, insurance	9	43.5	25.8	50.2	45.3	9.1	19.9	160.0	45.3	550.1	119.9	224.2	1293.4	1530.5	174.3	2998.3
Business services	10	23.3	78.7	135.5	136.6	10.4	81.7	238.2	89.6	206.3	266.3	312.1	1578.7	186.1	254.3	2019.1
Personal services	11	6.7	10.0	19.9	13.0	1.8	8.2	44.9	23.5	40.0	45.4	163.8	377.2	1699.3	1429.7	3506.2
Total		262.2	647.8	1143.6	1074.0	134.8	450.4	941.4	516.0	1041.1	735.3	1384.8	8331.2	6769.0	3138.6	18238.9
Wages		70.0	162.9	373.2	406.3	43.0	287.9	957.7	338.9	526.7	960.6	1633.1	5760.3			
Profits		145.3	128.1	274.2	136.6	150.8	118.5	665.6	286.3	1430.5	323.2	488.4	4147.4			
Output		477.5	938.7	1790.9	1616.9	328.6	856.8	2564.7	1141.2	2998.3	2019.1	3506.2	18238.9			

Price Valuation: Producers' prices
Currency: Bilions of $

Table 5.5: Input-Output Coefficients and Factor Elasticities of Austria in 2000

Sector	1	2	3	4	5	6	7	8	9	10	11
1	0.193	0.132	0.063	0.001	0.118	0.016	0.008	0.002	0.001	0.000	0.003
2	0.052	0.212	0.023	0.010	0.003	0.031	0.038	0.007	0.003	0.004	0.017
3	0.069	0.092	0.320	0.157	0.019	0.174	0.057	0.051	0.016	0.045	0.045
4	0.029	0.016	0.020	0.316	0.027	0.047	0.034	0.048	0.005	0.014	0.021
5	0.015	0.014	0.019	0.007	0.294	0.003	0.012	0.009	0.010	0.005	0.018
6	0.011	0.008	0.005	0.004	0.008	0.073	0.009	0.016	0.076	0.003	0.024
7	0.053	0.065	0.042	0.060	0.025	0.047	0.063	0.083	0.009	0.021	0.026
8	0.024	0.033	0.047	0.020	0.020	0.017	0.041	0.201	0.010	0.021	0.020
9	0.035	0.034	0.035	0.032	0.030	0.041	0.095	0.051	0.151	0.074	0.052
10	0.018	0.040	0.043	0.044	0.019	0.040	0.064	0.059	0.035	0.186	0.042
11	0.013	0.010	0.019	0.005	0.004	0.003	0.013	0.008	0.025	0.025	0.055

Table 5.6: Input-Output Coefficients and Factor Elasticities of the USA in 2000

Sector	1	2	3	4	5	6	7	8	9	10	11
1	0.181	0.145	0.094	0.001	0.200	0.007	0.005	0.004	0.001	0.002	0.004
2	0.031	0.163	0.012	0.011	0.002	0.045	0.035	0.003	0.008	0.004	0.014
3	0.080	0.112	0.269	0.146	0.031	0.151	0.040	0.063	0.019	0.042	0.055
4	0.020	0.015	0.024	0.274	0.012	0.056	0.028	0.038	0.005	0.018	0.027
5	0.014	0.012	0.017	0.005	0.007	0.003	0.011	0.005	0.008	0.006	0.013
6	0.002	0.002	0.002	0.002	0.012	0.001	0.004	0.003	0.009	0.004	0.013
7	0.040	0.080	0.066	0.081	0.014	0.106	0.041	0.044	0.016	0.033	0.033
8	0.026	0.037	0.040	0.025	0.067	0.028	0.032	0.152	0.016	0.042	0.036
9	0.091	0.028	0.028	0.028	0.028	0.023	0.062	0.040	0.183	0.059	0.064
10	0.049	0.084	0.076	0.084	0.032	0.095	0.093	0.078	0.069	0.132	0.089
11	0.014	0.011	0.011	0.008	0.006	0.010	0.017	0.021	0.013	0.022	0.047

Table 5.7: Leontief Inverse of Austria in 2000

Sector	1	2	3	4	5	6	7	8	9	10	11
1	1.278	0.242	0.141	0.046	0.225	0.064	0.038	0.027	0.016	0.016	0.026
2	0.099	1.303	0.064	0.044	0.031	0.065	0.065	0.029	0.015	0.016	0.034
3	0.193	0.256	1.549	0.392	0.112	0.342	0.149	0.162	0.077	0.114	0.115
4	0.079	0.069	0.075	1.496	0.083	0.102	0.075	0.112	0.026	0.040	0.050
5	0.041	0.044	0.053	0.033	1.431	0.023	0.030	0.029	0.023	0.017	0.036
6	0.029	0.029	0.025	0.022	0.026	1.093	0.028	0.036	0.101	0.018	0.038
7	0.104	0.135	0.105	0.132	0.073	0.094	1.099	0.139	0.029	0.046	0.051
8	0.067	0.091	0.112	0.078	0.060	0.060	0.079	1.280	0.029	0.048	0.044
9	0.094	0.111	0.109	0.110	0.089	0.101	0.155	0.121	1.203	0.128	0.092
10	0.069	0.113	0.118	0.129	0.066	0.100	0.118	0.130	0.069	1.254	0.080
11	0.030	0.031	0.043	0.026	0.017	0.020	0.028	0.024	0.036	0.040	1.067

Table 5.8: Leontief Inverse of the USA in 2000

Sector	1	2	3	4	5	6	7	8	9	10	11
1	1.260	0.256	0.182	0.052	0.265	0.059	0.032	0.030	0.015	0.021	0.031
2	0.057	1.219	0.037	0.034	0.020	0.070	0.051	0.015	0.017	0.013	0.027
3	0.186	0.264	1.442	0.327	0.107	0.277	0.102	0.146	0.058	0.098	0.124
4	0.056	0.060	0.070	1.408	0.041	0.106	0.056	0.079	0.020	0.042	0.057
5	0.026	0.028	0.033	0.019	1.016	0.014	0.017	0.013	0.013	0.012	0.020
6	0.006	0.007	0.007	0.006	0.015	1.005	0.007	0.007	0.012	0.007	0.016
7	0.088	0.151	0.130	0.161	0.050	0.160	1.072	0.083	0.036	0.061	0.066
8	0.069	0.098	0.097	0.080	0.105	0.073	0.063	1.206	0.037	0.073	0.069
9	0.174	0.115	0.105	0.101	0.087	0.083	0.110	0.091	1.246	0.106	0.114
10	0.132	0.200	0.183	0.208	0.092	0.188	0.155	0.154	0.120	1.193	0.154
11	0.031	0.032	0.031	0.028	0.018	0.026	0.029	0.036	0.023	0.034	1.060

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