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Report Paper

by

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Abstract

We investigate heterotic string models with $\mathcal{N} = (0, 2)$ worldsheet supersymmetry and GUT gauge group $SO(10)$. In this context we discuss the generalization of the heterotic (0,2) CFT/geometry correspondence [2,3] to the non-rational realm as proposed in [4, 5]. On the conformal field theory (CFT) side we construct Gepner-type $(0, 2)$ models using the powerful simple current formalism developed in [6, 7] and their relation to orbifolds with discrete torsion. [8] On the geometry side this construction is conjectured to correspond to non-linear sigma models on Calabi-Yau manifolds, known as Disterl-Kachru models [9]. In order to test the non-rational extension of the heterotic $(0, 2)$ CFT/geometry correspondence we develop tools for computing the charged massless non-singlet matter spectrum that allow us to count the number of generations, antigenerations and vectors for several rational and non-rational models and compare our predictions with the results from elliptic genera of Distler-Kachru models.

1 Introduction

The particle content and gauge interactions of our observable four dimensional world are quite accurately described by the Standard Model. However, it does not explain the values of its many parameters, such as masses and charges of the particles and the coupling constants of the forces governing their interactions. Why is it, for example, that the electron and the proton appear to have exactly the same electric charge up to a sign? It would be a remarkable coincidence if this was not due to some larger symmetry group, in which strong and electroweak interactions are embedded, that *predicts* charge quantization and the values of the charges of all elementary particles. This idea of *unifying* strong, weak and electromagnetic interaction into one single gauge group goes under the name of *Grand Unified Theory* (GUT). There are several models that implement this idea, some of which build on a supersymmetric extension of the Standard Model. A natural candidate of such a GUT is superstring theory, which is a ten dimensional supersymmetric theory, that has six dimensions *compactified* on an internal manifold as they are not directly observable. Superstring theory unifies strong and electroweak interactions but, moreover, it also includes gravity and, hence, serves even as a framework for quantum gravity. Despite its criticism, string theory is probably the most promising candidate for a Grand Unified Theory as it provides a much simpler and more predictive framework for a theory that underlies the known particle interactions. At the same time, it includes gravity into the picture, not by hand but it naturally arises in the spectrum as a spin 2 excitation of the string.

In 10 dimensions there are 5 different types of consistent superstrings (type I, type IIA, type IIB and two types of heterotic strings), which depend on whether they can break (open strings), the orientability of the worldsheet, the chirality of fermions, and the precise gauge group. Of particular interest are GUT gauge groups which, by construction, contain as a subgroup the Standard Model gauge group $SU(3) \times SU(2) \times U(1)$, where $SU(3)$ is the gauge group of strong interactions while the $SU(2) \times U(1)$ part describes electroweak interactions. In the context of GUTs the heterotic string is especially attractive as it already comes with a gauge group that contains the Standard Model quite naturally.

The heterotic string [1] is a closed string that is obtained by combining a superstring (right-moving) and a bosonic string (left-moving). While the bosonic string lives in 26 dimensions, the superstring exits only in 10 dimensions, so that the remaining 16 dimensions must be compactified on an internal space. Modular invariance of the partition function requires this space to be an even self-dual lattice of which there exist only two in 16 dimensions, the two being generated by the roots of $SO(32)$ and $E_8 \times E_8$, respectively. For phenomenolocigal reasons it is more interesting to study heterotic strings with gauge group $E_8 \times E_8$ as the Standard Model is embedded quite naturally. By breaking one of the E_8 factors one can study heterotic string models with GUT gauge groups E_6, or more realistically, $SO(10) \supset SU(5)$ and $SU(5) \supset SU(3) \times SU(2) \times U(1)$.
Our focus is on heterotic string models with gauge group $SO(10)$ and $\mathcal{N} = 2$ worldsheet supersymmetry and compute the spectrum of the charged massless matter in the theory using sophisticated conformal field theory (CFT) techniques. In this context the mechanism that breaks the gauge group of a $\mathcal{N} = (2,2)$ compactification from E_6 to

$E_5 \cong SO(10)$ is closely related to the breaking of supersymmetry in the left-moving sector, hence, leaving an unbroken $\mathcal{N} = (0,2)$ supersymmetric model. The tools we develop for the computation of the spectrum for this class of models can, in further work, be applied to models with smaller gauge groups like $E_4 \cong SU(5)$ or $E_3 \cong SU(3) \times SU(2)$. The motivation for this work lies in the conjectured heterotic $(0,2)$ CFT/geometry connection [2] and its extension to models with non-rational internal conformal field theories [4,5] which can be tested by comparing our results on the CFT side to the computation of the spectrum carried out by geometrical techniques.

In the search for viable string vacua, constructions that lead to $\mathcal{N} = 1$ spacetime supersymmetric models of elementary particle physics are of particular interest. These necessarily have their non-flat spacetime directions compactified on a Calabi-Yau manifold or, equivalently, the internal conformal field theory must have $\mathcal{N} = 2$ worldsheet supersymmetry.[1] During the past decades there has been particular focus on $\mathcal{N} = (2,2)$ theories, which, in the non-linear sigma model formulation, have the spin connection identified with the gauge connection breaking one of the E_8 factors down to E_6. This is still quite far from the Standard Model gauge group $SU(3) \times SU(2) \times U(1)$ but nevertheless they have been intensively studied since they give rise to three-generation models with small gauge group at the Planck scale. [2] In particular, after E.Witten's work [13] on the correspondence between non-linear sigma models on Calabi-Yau manifolds and orbifolds of Landau-Ginzburg models, the field of $(2,2)$ theories has undertaken a revival.

However, from the phenomenolocigal point of view, $(0,2)$ superconformal invariance is sufficient to realize observed particle physics features in the low-energy regime of heterotic string compactifications. These models are much less restrictive and, furthermore, lead to more realistic GUT gauge groups, like $SO(10)$ or $SU(5)$.
While a huge class of (0,2) models can be obtained by deformations of (2,2) theories, it has been pointed out in [9] that these models may only lie in a small subspace of the full moduli space. Instead, it seems that (0,2) models that are not merely deformations of their (2,2) counterparts make sense generically. Such vacua not only admit new quantum symmetries but also allow for a new kind of topology change. [9]

In [3] R. Blumenhagen and A. Wisskirchen have constructed a class of (0,2) models by generating new modular invariants from the class of Gepner models through a slight modification of the simple current method developped in [6, 7]. In [2] they conjecture, together with R. Schimmrigk, that this class of exactly solvable models is related to the linear sigma models considered in [9], which are known as Disterl-Kachru models, hence, extending the $(2,2)$ triality of exactly solvable models, Landau-Ginzburg (orbifold) models and (non-) linear sigma models on Calabi-Yau manifolds to the $(0,2)$ case. In the present work we investigate the generalization of this conjecture to the non-rational realm as proposed in [4, 5] by computing the spectrum of charged massless matter of a class of heterotic $(0,2)$ models with GUT gauge group $SO(10)$. Some of the results presented in this work will be published in [14].

[1] See, e.g. [3] and references therein.

2 Heterotic String Models

From a phenomenolocigal point of view, heterotic string models are very attractive since they already come with a gauge group that contains the Standard Model gauge group in a natural way. At first sight, there seems to be an obstacle arising from the assymetric nature of the heterotic sting because modular invariance, which is crucial in string theory, is only garanteed for left-right symmetric theories. However, there is an an elegant way around this obstacle, called Gepner map [15], which takes a consistent superstring theory to a consistent heterotic string theory. The inverse Gepner map can be used to map a bosonic string theory to a heterotic string theory. Both, the superstring and the bosonic string, are left-right symmetric and, hence, we can carry out all intermediate computations consistently in one of these framewokrs and, at the end of the day, use the (inverse) Gepner map to obtain a heterotic string compactification.

Rational Conformal Field Theory, Simple Currents and Minimal Models

Since we are interested in heterotic string compactifications that yield $\mathcal{N}=1$ spacetime supersymmetry we have to start from a $\mathcal{N}=2$ supersymmetric internal conformal field theory on the worldsheet of the string. In particular, we are interested in conformal field theories that have a finite number of primary fields and which are called *rational* conformal field theories (RCFT). *Simple currents*, which can be regarded as generalized free fields in rational conformal field theories, are primary fields that have a unique fusion product with all other primary fields. The universal center of $\mathcal{N}=2$ superconformal field theories (SCFT) of this type is constructed from two simple currents. Already for $\mathcal{N}=1$ superconformal symmetry the supercurrent is a simple current, which we denote by J_v. For $\mathcal{N}=2$ there exists an additional simple current, namely the spectral flow operator $J_s=e^{i\sqrt{c/12}\,X}$, which interpolates between the Ramond and the NS sector of the conformal theory. Under the spectral flow, $U(1)$ charges q and conformal weights h get shifted by

$$q=q_0+\frac{c}{6} \qquad \text{and} \qquad h=h_0+\frac{q_0}{2}+\frac{c}{24}, \tag{1}$$

where c denotes the central charge of the $\mathcal{N}=2$ SCFT. Since we want to construct Gepner-type models we need to introduce the notion of *minimal models*, which are particularly simple realizations of the $\mathcal{N}=2$ superconformal algebra (see appendix). Minimal models are quantum field theories constructed from a discrete series of rational unitary models with central charge $c=\frac{3k}{K}$, where k denotes the *level* of the minimal model and $K=k+2$ is the charge quantum in the NS sector. The primary fields of this theory, labeled by ϕ_m^{ls}, have conformal weights and $U(1)$ charges with respect to J_0 as

$$h_m^{ls}\equiv\frac{l(l+2)-m^2}{4K}+\frac{s^2}{8} \mod 1 \qquad \text{and} \qquad q_m^s\equiv\frac{s}{2}-\frac{m}{K} \mod 2. \tag{2}$$

Landau-Ginzburg description of Minimal Models

In [11] it has been conjectured that $\mathcal{N}=2$ minimal models in two dimensions are critical points of a superrenormalizable Landau-Ginzburg (LG) model, which captures critical phenomena in terms of effective Lagrangians. The Landau-Ginzburg superpotential for a minimal model is a Fermat-type polynomial $W(\phi)=\phi^K$. Hence, the minimal model factor in the tensor product is called *Fermat* factor $\mathcal{F}$. In order to obtain a spacetime supersymmetric string vacuum the $U(1)$ charges with respect to J_0 in the Landau-Ginzburg theory must be integral which can be achieved by modding out the symmetry $g=e^{2\pi i J_0}$. This procedure projects the $U(1)$ charges q to integral values and is called *orbifolding*. The minimal model is, hence, described by an Landau-Ginzburg orbifold model.

2.1 Heterotic E_6 Models

In [15] Gepner constructed heterotic string compactifications with gauge group E_6 from tensor products $\mathcal{C}_{int}=\bigotimes_i \mathcal{C}_{k_i}$ of minimal models $\mathcal{C}_{k_i}$ at level k_i and which are now called *Gepner models*. For heterotic $(0,2)$ compactifications with gauge group $SO(10)$ we can use a similar construction. In these Gepner-type models we use a tensor product of an arbitrary CFT, denoted by $\mathcal{C}'$, with a minimal model factor $\mathcal{F}$ at odd level $k=K-2$ that comprise the internal SCFT $\mathcal{C}_{int}=\mathcal{C}'\otimes\mathcal{F}$.
In order to be able to discuss heterotic $(0,2)$ models, let us review the structure of a generic four-dimensional compactification of the heterotic string. The right-moving sector consists of four spacetime coordinates and their superpartners $(X^\mu,\overline{\psi}^\mu)$, a ghost plus superghost system (b,c,β,γ), and an "internal" $\mathcal{N}=2$ superconformal field theory $\mathcal{C}_{int}$ with central charge $\overline{c}=9$, so that total central charge $\overline{c}_{tot}=4\cdot\frac{3}{2}-26+11+9=0$ is anomaly free. The left-moving sector is a bosonic string with spacetime plus ghost part (X^μ,b,c) and the same internal sector $\mathcal{C}_{int}$ so that a left-moving CFT with central charge 13 needs to be added to for anomaly cancellation $c_{tot}=4\cdot1-26+9+13=0$. Modular invariance requires this CFT to be either an $\hat{\mathfrak{so}}(10)\times\hat{\mathfrak{e}}_8$ or $\hat{\mathfrak{so}}(26)$ level 1 affine Lie algebra, where we will, henceforth, ignore the phenomenologically less attractive $\hat{\mathfrak{so}}(26)$. The representations of $\hat{\mathfrak{so}}(10)_1$ are the singlet $\mathbb{1}$, the vector v, the spinor s and conjugate spinor $\bar{\mathrm{s}}$ and the fusion rules are given by $\mathrm{sv}=\bar{\mathrm{s}}$, $\mathrm{s}^2=\bar{\mathrm{s}}^2=\mathrm{v}$ and $\mathrm{v}^2=\mathbb{1}$. Instead of this covariant quantization we can also use light-cone gauge, which amounts to ignoring the (super-) ghosts and restricting the spacetime coordinates to transverse directions.

A viable superstring vacuum is then obtained by aligning spacetime spinors and tensors with internal Ramond and Neveu-Schwarz sectors, respectively, by the means of the alignement current $J_{RNS}=J_v\otimes\mathrm{v}$ and carrying out the (generalized) GSO projection by the current $J_{GSO}=J_s\otimes\mathrm{s}$. The first factor denotes the internal $\mathcal{N}=2$ SCFT while the second factor denotes the $SO(10)$ representation. Both steps can be achieved by applying simple current techniques [6], for which the theory has to be cast into a left-right symmetric form. Therefore, we apply the Gepner map to the right-movers. The GSO projection, then, promotes the gauge group $SO(10)$ to E_6 and amounts to keeping only states with even $U(1)$ charges in the bosonic sector.

Starting from smaller building blocks and using simple current techniques we can break the gauge group E_6 of Gepner's construction in the left-moving sector to $SO(10)$ by the means of a simple current J_b, which we call the *Bonn twist*. At the same time, worldsheet supersymmetry is reduced from $(2,2)$ to $(0,2)$.

2.2 Heterotic $SO(10)$ Models

While $(2,2)$ models with $SO(10)$ gauge group can be constructed from a four dimensional bosonic string with internal CFT given by $\mathcal{C}_{int} \otimes \hat{\mathfrak{so}}(10) \times \hat{\mathfrak{e}}_8$ after the Gepner map, the internal CFT needs to be split into smaller building blocks for $(0,2)$ models in order to be able to break supersymmetry only in the left-moving sector. We thus decompose $\mathcal{C}_{int} = \mathcal{C}' \otimes \mathcal{F}$, where $\mathcal{C}'$ is a general CFT while $\mathcal{F}$ is the minimal model at odd level $k = K - 2$ introduced above. In the gauge sector we start with an $D_4 = SO(8)$ gauge group and extend it to $D_5 = SO(10)$ in the left-moving bosonic sector by adding an $D_1 = SO(2)$ factor, and to E_6 in the right-moving sector which amounts to spacetime supersymmetry after the inverse Gepner map back to the heterotic string. Our $(0,2)$ models with $SO(10)$ gauge group, hence, are constructed from a four dimensional bosonic string with an internal $c = 22$ CFT $\mathcal{C}' \otimes \mathcal{F} \otimes D_1 \otimes D_4 \times E_8$ with current algebras $\hat{\mathfrak{so}}(2n)_1$ and $(\hat{\mathfrak{e}}_8)_1$ and a certain simple current modular invariant that gives rise to alignment of spin structures and the generalized GSO projection. States in a $(0,2)$ model have the structure $\Phi_{(0,2)} = \phi_{\mathcal{C}'} \otimes \phi_{\mathcal{F}} \otimes \chi_{D_1} \otimes \chi_{D_4}$, where the E_8 part, that acts only as a spectator, has been omitted.

2.2.1 Simple Current Construction

The simple current modular invariant that defines our resulting $(0,2)$ models is based on the simple current group $\mathcal{G}$ generated by J_{GSO}, J_A, J_b, J_C with

$$J_{GSO} = J_s \otimes J_s \otimes s \otimes S, \qquad J_A = 1 \otimes 1 \otimes v \otimes V, \qquad J_C = J_v \otimes 1 \otimes 1 \otimes V \tag{3}$$

and the Bonn twist

$$J_b = 1 \otimes (J_s)^K \, (J_v)^{\frac{K-1}{2}} \otimes s \otimes 1, \tag{4}$$

where the decomposition is with respect to $\mathcal{C}' \otimes \mathcal{F} \otimes D_1 \otimes D_4$ with the E_8 spectator being omitted. The chiral algebra in the right-moving sector contains all possible alignment currents that align the different factors in the tensor product

$$\overbrace{\underbrace{\mathcal{C}' \otimes \mathcal{F}}_{J_b^2} \otimes \underbrace{D_1 \otimes D_4}_{J_A}}^{J_C}.$$

The chiral algebra in the left-moving sector lacks the alignment current J_A and instead contains the Bonn twist J_b. This is the reason why the gauge group in the left-moving sector is $SO(10)$ instead of E_6 and supersymmetry is broken from $(2,2)$ to $(0,2)$. The alignment of $D_4 \times D_1 \to D_5$ in the right-moving sector is achieved by J_A while the extension to $D_5 = SO(10)$ in the left-moving sector is carried out by J_{GSO}. The GSO current further extends $D_5 \to E_6$ in the right-moving sector which, after the map to the heterotic string, corresponds to odd integral $U(1)$ charges and $\mathcal{N} = 1$ spacetime supersymmetry.

3 Computation of the Charged Massless Spectrum

In this section we present the basic steps in order to compute the charged massless matter spectrum of our $SO(10)$ model. We will use these results in section 4 in order to test the $(0,2)$ CFT/geometry connection by comparing with spectra obtained by geometric methods.

3.1 Quantum Numbers for Chiral and Vector Multiplet

After the alignment-extension of $D_4 \times D_1$ to D_5 we can perform the Gepner map on the right-moving side $D_5 \to D_1 = SO(2)_{LC}$ to obtain spacetime quantum numbers (in light-cone gauge) from the representaions of D_5. The SUSY multiplets yielding spacetime matter and spacetime gauge symmetry generators are then assembled by J_{GSO}. Admissible states are selected by imposing the massless condition $\overline{h}_{tot} = \overline{h}_{st} + \overline{h}_{int} = \frac{1}{2}$ and the GSO projection on the bosonized string. Table 1 shows how the Gepner map G acts on the characters of D_5 to get the associated spacetime representation. From left to right we give the spacetime conformal weight and the internal quantum numbers which are obtained by charge selection rules and the unitarity bound $|\overline{q}_{int}| \leq \frac{\overline{c}}{6} = \frac{3}{2}$ for states in the Ramond sector and $|\overline{q}_{int}| \leq 2\overline{h}_{int}$ for states in the NS sector.

χ_{D_5}	$\overset{G}{\to}$	$\chi_{SO(2)_{LC}}$	$\overline{h}_{st}$	$\overline{h}_{int}$	$\overline{q}_{int}$	state
$\mathbb{1}$	$\to$	v	$\frac{1}{2}$	0	0	$\mathbb{1}$
v	$\to$	$\mathbb{1}$	0	$\frac{1}{2}$	± 1	**c**, **a**
s	$\to$	$-\overline{s}$	$\frac{1}{8}$	$\frac{3}{8}$	$-\frac{1}{2}$, $\frac{3}{2}$	$\mathbf{R_0}$
$\overline{\text{s}}$	$\to$	$-s$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{2}$, $-\frac{3}{2}$	$\mathbf{R_0}$

Table 1: Right-moving states with internal and spacetime quantum numbers

Since on the right-moving side we have full RNS alignment the $SO(2)_{LC}$ representations are paired with internal states of the same sector. From the condition for massless states and the unitarity bound it follows that the only admissible internal states are BPS states. In the NS sector the internal states that fullfill the BPS condition $\overline{h}_{int} = |\frac{\overline{q}_{int}}{2}|$ are chiral and antichiral states, denoted by **c** and **a**. In the Ramond sector the internal states that satisfy the analogous unitarity bound are Ramond ground states since $\overline{h}_{int} = \frac{\bar{c}}{24} = \frac{3}{8}$ and are denoted by $\mathbf{R_0}$.

The quantum numbers $(\bar{h}_{int}, \bar{q}_{int})$ for the massless SUSY multiplets are:

- Chiral multiplets: fermions $(3/8, -1/2)$, scalars $(1/2, 1)$.
- Antichiral multiplets: fermions $(3/8, 1/2)$, scalars $(1/2, -1)$.
- Vector multiplets: gauge bosons $(0, 0)$ and left/right-handed gauginos $(3/8, \pm 3/2)$.

3.2 Counting Massless States

In order to count massless states we need to specify a right-moving representative on the orbit of which we determine admissible left-moving states. Furthermore, wen need to choose a representation of the $SO(10)$ gauge group under which the left-moving (bosonic) states transform. In the language of Distler-Kachru [9] and of Blumenhagen-Wißkirchen [2,3], the relevant $D_5 = SO(10)$ decompositions under the maximal subgroup $SO(8) \times U(1)$ are identified as

$$\begin{array}{lcccccc} \mathbf{1} & = & & & \mathbf{1_0} & & \\ \mathbf{10} & = & \mathbf{1_{-2}} & \oplus & \mathbf{8^s_0} & \oplus & \mathbf{1_2} \\ \mathbf{16} & = & & \mathbf{8^v_{-1}} & \oplus & \mathbf{8^{\bar{s}}_1} & \\ \overline{\mathbf{16}} & = & & \mathbf{8^v_1} & \oplus & \mathbf{8^{\bar{s}}_{-1}} & \end{array} . \tag{5}$$

The notation is $\mathbf{N}^{\chi}_{\widetilde{q}}$, where $\mathbf{N}$ is the dimension of the D_4 representation, χ denotes the $SO(8)$ character and $\widetilde{q} = q_{int} + q^{D_1}$ is the $U(1)$ charge associated with the $U(1)$ current of the $SO(10) \supset SO(8) \times U(1)$ decomposition that is a linear combination of the $U(1)$ currents of the $\mathcal{N} = 2$ algebra of C_{int} and of $SO(2) = D_1$.

In the right-moving sector the structure of massless states is highly constrained due to the RNS alignment following from supersymmetry while in the left-moving sector, where this alignment is partially broken, a variety of states is admitted. We use the restricted structure in the right-moving sector and construct admissible left-moving states on orbits of admissible right-moving states, the pairings of which give the massless spectrum of the heterotic $(0, 2)$ string.
Admissible left-moving states are obtained by twisting admissible right-moving states by $J = J^{\nu}_{GSO} J^{\alpha}_{A} J^{\beta}_{b} J^{\gamma}_{C}$ with $\alpha, \gamma = 0, 1$, $\beta = 0, 1, 2, 3$ and $\nu = 0, ..., 2M' - 1$ and imposing the condition for massless states $h_{tot} = h' + h^{\mathcal{F}} + h^{D_1} + h^{D_4} = 1$ in the bosonic sector. A generic left-moving state is obtained by a generic right-moving state by

$$|\mathcal{C}' \otimes \mathcal{F} \otimes D_1 \otimes D_4 >_l = J^{\nu}_{GSO}\, J^{\alpha}_{A}\, J^{\beta}_{b}\, J^{\gamma}_{C}\, |\mathcal{C}' \otimes \mathcal{F} \otimes D_1 \otimes D_4 >_r \tag{6}$$

and the explicit form of the twist current is given by

$$J \;=\; J^{\nu}_{s} J^{\gamma}_{v} \;\otimes\; J^{\nu+\beta K}_{s} J_{v}^{\frac{K-1}{2}\beta} \;\otimes\; s^{\nu+\beta} v^{\alpha} \;\otimes\; S^{\nu} V^{\alpha+\gamma}. \tag{7}$$

Besides organizing the contributions to the spectrum in twisted sectors the exponents $\nu, \alpha, \beta, \gamma$ determine whether a left-moving (twisted) sector yields the same field as the right-moving sector on the orbit of which it is computed or its superpartner. By choosing a specific SUSY multiplet together with an $SO(10)$ represenation for the gauge multiplet we study the structure of the charged massless spectrum of non-singlet matter states. We use the information obtained from the exact CFT calculations to determine the number of generations, antigenerations and vectors by the means of the *extended Poincaré polynomial* and the *complementary Poincaré polynomial*.

3.2.1 Extended Poincaré Polynomial

The extended Poincaré polynomial (EPP) of an $\mathcal{N}=2$ SCFT as given by [12]

$$\mathcal{P}(_{(c,c)}t,\bar{t},x)=\sum_{l\geq 0}\sum_{\kappa=0,1}x^{l}\,(-1)^{\kappa}\,P_{l,\kappa}(t,\bar{t}), \tag{8}$$

is the sum of $J_s^{2l}J_v^{\kappa}$–twisted Poincaré Polynomials weighted by an additional sign, that is related to the ambiguity of dealing with a field or its superpartner, i.e. a possible application of the supercurrent J_v. The ordinary Poincaré Polynomial is given by

$$\mathcal{P}_{l,\kappa}(t,\bar{t})=\sum_{\substack{(a,\bar{a})\in\mathcal{R}_{(c,c)}\\ \bar{a}=J_s^{2\,l}\,J_v^{\kappa}\,a}}t^{q(a)}\,\bar{t}^{\bar{q}(\bar{a})}, \tag{9}$$

where t and $\bar{t}$ can be regarded as independent variables and the sum is over states in the (c,c) ring. In the case where the internal sector has aligned spin structures (corresponding to a twist by an even exponent of J_b) the states contributing to the massless spectrum are BPS states. We can determine the number of aligned generations, antigenerations and vectors by looking for particular terms in the EPP that are determined by the $U(1)$ charges of the internal left- and right-moving sector.

3.2.2 Complementary Poincaré Polynomial

In the case where the internal sector has non-aligned spin structures (odd exponent of J_b) also non-BPS states can contribute to the massless spectrum and we thus need in addition to the information of the (left-moving) internal $U(1)$ charge also the conformal weight. We are thus interested in the complementary Poincaré polynomial (CPP) [14]

$$\mathscr{P}(x,q,t)=\sum_{l\geq 0}\sum_{\kappa=0,1}\sum_{\substack{\bar{a}\in\mathcal{R}_0\\ a=J_s^{2l}J_v^{\kappa}\bar{a}}}(-1)^{\kappa}\,x^{l}\,q^{(L_0-\frac{c'}{24})(a)}t^{J_0(a)}, \tag{10}$$

where q and t are independent variables. This polynomial is complementary to the EPP. It does not involve the right-mover's charge, but instead keeps track of the conformal dimension of excited left-moving states. We can determine the number of non-aligned generations, antigenerations and vectors by looking for particular terms in the CPP determined by the left-moving conformal weight and $U(1)$ charge.

3.2.3 Counting Generations

We have now assembled all tools to compute the charged massless spectrum. Unitarity bounds and the condition for massless states restrict the possible states in the left-moving sector. By choosing a right-moving representative we can determine the admissible left-moving states on its orbit. In a careful analysis with separate discussion of even ("aligned") and odd ("non-aligned") exponent of the Bonn twist the structure of admissible states and, hence, admissible terms in the EPP and CPP can be determined. All admissible terms in $\mathcal{P}_{(c,c)}(t,\bar{t},x)$ of $\mathcal{C}'$ for aligned generations are summarized in table 2. All admissible terms in $\mathscr{P}(x,q,t)$ of $\mathcal{C}'$ for non-aligned generations are summarized in table 3.

16 - aligned generations				
σ'	$\bar{\ell}$	l	$\bar{q}'_{\mathbf{c}}$	$q'_{\mathbf{c}}$
$+$	$\bar{\ell} \in 2\mathbb{Z}, \quad 0 \leq \bar{\ell} \leq k$	$l \in 2K\mathbb{Z}$ $l \in 2K\mathbb{Z}+1$	$\frac{K+2+\bar{\ell}}{K}$	$\frac{K+2+\bar{\ell}}{K}$
$-$	$\bar{\ell} \notin 2\mathbb{Z}, \quad 0 \leq \bar{\ell} \leq k$	$l \in 2K\mathbb{Z}+1$ $l \in 2K\mathbb{Z}$	$\frac{K+2+\bar{\ell}}{K}$	$\frac{2K-\bar{\ell}}{K}$

Table 2: Aligned generations: Left- and right-moving $\mathcal{C}'$-sector charges $q'_{\mathbf{c}}$ and $\bar{q}'_{\mathbf{c}}$, right-moving label $\bar{\ell}$, exponent l of x and sign $\sigma' = (-1)^{\gamma}$ in the EPP of $\mathcal{C}'$ where terms are proportional to $\sigma' x^l \, t^{q'_{\mathbf{c}}} \, \bar{t}^{\bar{q}'_{\mathbf{c}}}$.

As the analysis for antigenerations and vectors goes along the same lines and yields similar tables we will not go into details here.

16 - non-aligned generations							
Fermat	$K \mod 4$	σ'	$\overline{\ell}$	m	$\nu \mod 4K$	h'_R	q'_R
$\varphi^{\overline{\ell},1}_{\overline{\ell}+1}$	1	$-$	$\overline{\ell} \notin 2\mathbb{Z}, \quad \frac{K-3}{2} \leq \overline{\ell} \leq k$	$m = \overline{\ell}+1$	$1-\beta K$	$\frac{\overline{\ell}+2}{2K}$	$\frac{\overline{\ell}+2}{K}$
	3	$-$	$\overline{\ell} \in 2\mathbb{Z}, \quad \frac{K-3}{2} \leq \overline{\ell} \leq k$	$m = \overline{\ell}+1$	$1-\beta K+2K$	$\frac{\overline{\ell}+2}{2K}$	$\frac{\overline{\ell}+2}{K}$
$\varphi^{\overline{\ell},-1}_{-(\overline{\ell}+1)}$	1	$+$	$\overline{\ell} \in 2\mathbb{Z}, \quad 0 \leq \overline{\ell} \leq \frac{K-1}{2}$	$m = -(\overline{\ell}+1)$	$-1-2\overline{\ell}-\beta K$	$\frac{K-\overline{\ell}}{2K}$	$\frac{K-\overline{\ell}}{K}$
	3	$+$	$\overline{\ell} \notin 2\mathbb{Z}, \quad 0 \leq \overline{\ell} \leq \frac{K-1}{2}$	$m = -(\overline{\ell}+1)$	$-1-2\overline{\ell}-\beta K$	$\frac{K-\overline{\ell}}{2K}$	$\frac{K-\overline{\ell}}{K}$
$\varphi^{\overline{\ell},-1}_{m}$	1	$+$	$\overline{\ell} \in 2\mathbb{Z}, \quad 0 \leq \overline{\ell} \leq k$	$m \notin 2\mathbb{Z}, \quad \lvert m\rvert < \overline{\ell}$ $(\lvert m\rvert-1)^2 \geq \overline{\ell}(\overline{\ell}+2)+1-K$	$(K-1)(\overline{\ell}-m-1)-1-\beta K$	$\frac{3K+2}{8K} - h^{\overline{\ell},-1}_{m} - \frac{1}{2}q^{\overline{\ell},-1}_{m}$	$\frac{K+2}{2K} - q^{\overline{\ell},-1}_{m}$
	3	$+$	$\overline{\ell} \notin 2\mathbb{Z}, \quad 0 \leq \overline{\ell} \leq k$	$m \in 2\mathbb{Z}, \quad \lvert m\rvert < \overline{\ell}$ $(\lvert m\rvert-1)^2 \geq \overline{\ell}(\overline{\ell}+2)+1-K$	$-(K+1)(\overline{\ell}-m+1)+1-\beta K$	$\frac{3K+2}{8K} - h^{\overline{\ell},-1}_{m} - \frac{1}{2}q^{\overline{\ell},-1}_{m}$	$\frac{K+2}{2K} - q^{\overline{\ell},-1}_{m}$
$\varphi^{\overline{\ell},1}_{m}$	1	$-$	$\overline{\ell} \notin 2\mathbb{Z}, \quad 0 \leq \overline{\ell} \leq k$	$m \in 2\mathbb{Z}, \quad \lvert m \pm K\rvert \leq K-2-\overline{\ell}$ $(\lvert m \pm K\rvert-1)^2 \geq (K-2-\overline{\ell})(K-\overline{\ell})+1-K$	$(K-1)(\overline{\ell}-m+1)+1-\beta K$	$\frac{3K+2}{8K} - h^{K-2-\overline{\ell},-1}_{m\pm K} - \frac{1}{2}q^{K-2-\overline{\ell},-1}_{m\pm K}$	$\frac{K+2}{2K} - q^{K-2-\overline{\ell},-1}_{m\pm K}$
	3	$-$	$\overline{\ell} \in 2\mathbb{Z}, \quad 0 \leq \overline{\ell} \leq k$	$m \notin 2\mathbb{Z}, \quad \lvert m \pm K\rvert \leq K-2-\overline{\ell}$ $(\lvert m \pm K\rvert-1)^2 \geq (K-2-\overline{\ell})(K-\overline{\ell})+1-K$	$(K+1)(-\overline{\ell}+m+1)-1-\beta K$	$\frac{3K+2}{8K} - h^{K-2-\overline{\ell},-1}_{m\pm K} - \frac{1}{2}q^{K-2-\overline{\ell},-1}_{m\pm K}$	$\frac{K+2}{2K} - q^{K-2-\overline{\ell},-1}_{m\pm K}$

Table 3: Non-aligned generations: Left-moving $\mathcal{C}'$-sector charges q'_R and conformal weights h'_R (in the Ramond sector), signs $\sigma' = (-1)^\gamma$ and constraints for the admissible terms in the CPP in $\mathcal{C}' \sim \sigma' x^{\nu/2} q^{h'_R - \frac{c'}{24}} t^{q'_R}$.

4 Testing the $(0,2)$ CFT/Geometry Connection

In the following we will illustrate our results from the previous section in two examples and compare our predictions with that of Blumenhagen-Wißkirchen and Distler-Kachru. As a prominent example of Fermat-type models we show that the number of generations, antigenerations and vectors of the (0,2) cousin of the quintic as computed on the CFT side by our counting algorithm agrees with those calculated in [3]. In a depicted example of a non-Fermat-type LG model the numbers of generations and antigenerations we predict on the CFT side agree with that computed by the χ-genus of Disterl-Kachru models [9].

4.1 Fermat-type Landau-Ginzburg models

We consider the (0,2) cousin of the quintic, which is a tensor product of five minimal models with levels $(k'_1, k'_2, k'_3, k'_4; k) = (3,3,3,3;3)$, where the first four entries comprise $\mathcal{C}'$ which, in this Fermat-type LG model, is a tensor product of minimal models, and the last entry corresponds to the Fermat factor $\mathcal{F}$. The results of the analysis carried out in [2,3] is given below.

$N_{\mathbf{16}} = N^{\mathrm{A}}_{\mathbf{16}} + N^{\mathrm{NA}}_{\mathbf{16}}$	$N_{\overline{\mathbf{16}}} = N^{\mathrm{A}}_{\overline{\mathbf{16}}} + N^{\mathrm{NA}}_{\overline{\mathbf{16}}}$	$N_{\mathbf{10}} = (N^{\mathrm{A}_2}_{\mathbf{10}} + N^{\mathrm{A}_1}_{\mathbf{10}}) + N^{\mathrm{NA}}_{\mathbf{10}}$
$80 = 60 + 20$	0	$74 = (41+1) + 32$

Shown are the number of generations $N_{\mathbf{16}}$, antigenerations $N_{\overline{\mathbf{16}}}$ and vectors $N_{\mathbf{10}}$ for the model $3^4 \otimes 3$, with the superscripts A = aligned, AN = non-aligned; A_1 = aligned with $q_{int} = 1$, A_2 = aligned with $q_{int} = 2$.

In order to derive the spectrum ($N_{\mathbf{16}} = 80$, $N_{\overline{\mathbf{16}}} = 0$, $N_{\mathbf{10}} = 74$) of the (0,2) cousin of the quintic using our counting algorithm, we decompose the "quintic Gepner model" 3^5 into $\mathcal{C}' = 3^4$ and an additional Fermat factor Φ^5, i.e. a minimal model at level $k = 3$, on which the Bonn-twist acts [5].
The relevant data as obtained from our counting algorithm can be summarized by the following tables.

16 - aligned generations				
σ'	$\overline{\ell}$	l	$\overline{q}'_{\mathbf{c}}$	$q'_{\mathbf{c}}$
$+$	0	0	$\frac{7}{5}$	$\frac{7}{5}$
$+$	2	0	$\frac{9}{5}$	$\frac{9}{5}$

16 - non-aligned generations						
Fermat	$K \mod 4$	σ'	$\overline{\ell}$	$\nu \mod 4K$	h'_R	q'_R
$\varphi^{\overline{\ell},-1}_{-(\overline{\ell}+1)}$	1	$+$	2	10	$\frac{3}{10}$	$\frac{3}{5}$

10 - aligned vectors					
q_{int}	σ'	$\bar{\ell}$	l	$\bar{q}'_{\mathbf{c}}$	$q'_{\mathbf{c}}$
2	+	1	0	$\frac{8}{5}$	$\frac{8}{5}$
2	+	3	0	$\frac{10}{5}$	$\frac{10}{5}$
1	+	1	2	$\frac{8}{5}$	$\frac{4}{5}$

10 - non-aligned vectors							
Fermat	K mod 4	σ'	$\bar{\ell}$	m	ν mod $4K$	h'_R	q'_R
$\varphi^{\bar{\ell},-1}_{-(\bar{\ell}+1)}$	1	+	1	-2	2	$\frac{19}{10}$	$\frac{14}{5}$
$\varphi^{\bar{\ell},1}_{m}$	1	+	1	4	8	$\frac{11}{10}$	2

We encode the charge degeneracies of the GSO-twisted but unprojected $\mathcal{N}=2$ SCFT $\mathcal{C}'$, with alignment between $\mathcal{C}'$ and the Fermat factor, in the extended Poicaré polynomial [12] and the Complementary Poincaré polynomial. For the untwisted sector we obtain the standard Poincaré polynomial (in the (c,c) ring)

$$\begin{aligned} P(t,\bar{t}) &= \tfrac{(1-T^4)^4}{(1-T)^4} = (1+T+T^2+T^3)^4 \;=\; 1+4T+10T^2+20T^3+31T^4+ \\ &\quad +40T^5+44T^6+40T^7+31T^8+20T^9+10T^{10}+4T^{11}+T^{12} \end{aligned} \tag{11}$$

with $T=(t\bar{t})^{1/5}$. In the twisted sectors only the ground states contribute since there are no invariant fields. Hence the EPP continues with the terms

$$P(x,t^5,\bar{t}^5) = P(t^5,\bar{t}^5) + x\,\bar{t}^{\,12} + x^2\,t^4\bar{t}^{\,8} + x^3\,t^8\bar{t}^{\,4} + x^4\,t^{12} + \ldots \tag{12}$$

and then "periodically" with $x^5 P(t^5,\bar{t}^5) + x^6\,\bar{t}^{\,12} + \ldots$
In order to determine the number of aligned generations, antigenerations and vectors for the (0,2) cousin we read off the relevant data from the tables above to get $N^A_{\mathbf{16}}=60$ from the T^7 and T^9 terms, $N^A_{\overline{\mathbf{16}}}=0$ and $N^A_{\mathbf{10}}=74$ from the T^8 and T^{10} terms in (11) and from the $x^2t^4\bar{t}^8$ term in (12).

The Complementary Poincaré polynomial $\mathscr{P}(x,q,t)$ reads (up to $\mathcal{O}(q^{8/5})$ terms)

$$
\begin{aligned}
\mathscr{P}(x,q,t^5) &= \frac{1}{t^6}+\frac{4}{t^5}+\frac{10}{t^4}+\frac{20}{t^3}+\frac{31}{t^2}+\frac{40}{t}+44+40t+31t^2+\mathbf{20t^3}+\\
&+ 10t^4+4t^5+t^6+\\
&+ x\Bigg[t^6+q^{1/5}\left(-4t^2+4t^7\right)+q^{2/5}\left(\frac{6}{t^2}-16t^3+10t^8\right)+q^{3/5}\Big(\\
&\quad -\frac{4}{t^6}+\frac{24}{t}-40t^4+20t^9\Big)+q^{4/5}\Big(60+\frac{1}{t^{10}}-\frac{16}{t^5}-76t^5+\\
&\quad +31t^{10}\Big)+\cdots+q^{8/5}\left(\frac{4}{t^{11}}-\frac{57}{t^6}+\frac{168}{t}-150t^4+4t^9+\mathbf{31t^{14}}\right)\Bigg]\\
&+ x^2\Bigg[t^2+q^{2/5}\left(-\frac{4}{t^2}+4t^3\right)+q^{3/5}\left(4t-4t^6\right)+q^{4/5}\Big(\frac{6}{t^6}-\frac{16}{t}+\\
&\quad +10t^4\Big)+\cdots+q^{8/5}\left(\frac{1}{t^{14}}-\frac{16}{t^9}+\frac{20}{t^4}+28t-57t^6+24t^{11}\right)\Bigg]\\
&+ x^3\Bigg[\frac{1}{t^2}+q^{2/5}\left(\frac{4}{t^3}-4t^2\right)+q^{3/5}\left(-\frac{4}{t^6}+\frac{4}{t}\right)+q^{4/5}\Big(\frac{10}{t^4}-16t+\\
&\quad +6t^6\Big)+\cdots+q^{8/5}\left(\frac{24}{t^{11}}-\frac{57}{t^6}+\frac{28}{t}+20t^4-16t^9+t^{14}\right)\Bigg]\\
&+ x^4\Bigg[\frac{1}{t^6}+q^{1/5}\left(\frac{4}{t^7}-\frac{4}{t^2}\right)+q^{2/5}\left(\frac{10}{t^8}-\frac{16}{t^3}+6t^2\right)+q^{3/5}\Big(\frac{20}{t^9}-\\
&\quad -\frac{40}{t^4}+24t-4t^6\Big)+q^{4/5}\left(60+\frac{31}{t^{10}}-\frac{76}{t^5}-16t^5+\mathbf{t^{10}}\right)+\\
&\quad \cdots+q^{8/5}\left(\frac{31}{t^{14}}+\frac{4}{t^9}-\frac{150}{t^4}+168t-57t^6+4t^{11}\right)\Bigg]+\cdots, \qquad (13)
\end{aligned}
$$

with the next terms being "periodic" in x. The number of non-aligned generations, antigenerations and vectors is read off from 13 using the information of the tables above. We get $N^{NA}_{\mathbf{16}} = 20$ from the q^0t^3 term and $N^{NA}_{\mathbf{10}} = 32$ from the coefficients of $xq^{8/5}t^{14}$ and $x^4q^{4/5}t^{10}$. These numbers agree with those obtained in [3].

4.2 Non-Fermat-type Landau-Ginzburg models

We consider the model $\mathbb{P}_{1,2,2,3,2}[10]$, which has $K = 5$ and can therefore be used for checking the case where $K \equiv 1 \mod 4$. The CFT/geometry conjecture predicts equivalence with the Disterl-Kachru model

$$V_{1,2,2,3,2}[10] \longrightarrow \mathbb{P}_{1,2,2,3,4,4}[8,8]. \qquad (14)$$

Its χ-genus can be computed to yield

$$
\begin{array}{|c|c|}
\hline
\alpha & \chi_\alpha \\
\hline
0 & -t^4 - 55t^3 + 55t + 1 \\
1 & t^4 \\
2 & -2t^2 \\
3 & t^2 + t \\
4 & -t \\
5 & 5t - 5t^3 \\
6 & t^3 \\
7 & -t^3 - t^2 \\
8 & 2t^2 \\
9 & -1 \\
\hline
\end{array}
\tag{15}
$$

Summing up the positive coefficients of monomials in t and t^3, respectively, we get 61 generations and 1 antigeneration for the Disterl-Kachru model. By using our counting method, we can compare this prediction with our results on the CFT side. The relevant data for admissible terms in the EPP of $\mathcal{C}'$, counting aligned generations, is listed in the table below. Using the extended Poincaré polynomial and the complementary Poincaré polynomial (wich are quite lengthy and will, thus, not give here) we can determine the number of generations, antigenerations and vectors as follows.

16 - aligned generations					
σ'	$\bar{\ell}$	l	$\bar{q}'_{\mathbf{c}}$	$q'_{\mathbf{c}}$	$N^A_{\mathbf{16}}$
$+$	0	0	$\frac{7}{5}$	$\frac{7}{5}$	**27**
$+$	2	0	$\frac{9}{5}$	$\frac{9}{5}$	**14**
$+$	0	5	$\frac{7}{5}$	$\frac{7}{5}$	**3**
$+$	2	5	$\frac{9}{5}$	$\frac{9}{5}$	**1**

Hence, the number of aligned generation is $27+14+3+1 = 45$. The necessary information in order to count non-aligned generation in the complementary Poincaré polynomial is given by

16 - non-aligned generations							
Fermat	$K \mod 4$	σ'	$\bar{\ell}$	$\nu \mod 4K$	h'_R	q'_R	$N^{NA}_{\mathbf{16}}$
$\varphi^{\bar{\ell},-1}_{-(\bar{\ell}+1)}$	1	$+$	0	14	$\frac{1}{2}$	1	**1**
$\varphi^{\bar{\ell},-1}_{-(\bar{\ell}+1)}$	1	$+$	2	10	$\frac{3}{10}$	$\frac{3}{5}$	**1**
$\varphi^{\bar{\ell},-1}_{-(\bar{\ell}+1)}$	1	$+$	2	0	$\frac{3}{10}$	$\frac{3}{5}$	**14**

There are $1+1+14=16$ non-aligned vector which together with the 45 aligned vectors give a total of 61 vectors which agrees with the prediction from the Disterl-Kachru model. In order to count aligned antigenerations we need the following data.

$\overline{16}$ - **aligned antigenerations**					
σ'	$\bar{\ell}$	l	$\bar{q}'_{\mathbf{c}}$	$q'_{\mathbf{c}}$	$N^{A}_{\overline{\mathbf{16}}}$
$+$	0	6	1	$\frac{7}{5}$	**1**

Since there are no non-aligned antigenerations there is only 1 antigeneration which is in agreement with the predition of the Disterl-Kachru model.

5 Conclusion and Outlook

We have investigated Gepner-type models with internal $\mathcal{N}=(0,2)$ superconformal symmetry and GUT gauge group $SO(10)$ constructed by the means of the simple current formalism. In this context we have derived a counting algorithm that enables us to calculate the charged massless matter spectrum for a broad class of models. This algorithm is based on the extended Poincaré polynomial and the complementary Poincaré polynomial which encode the information of charge degeneracies and conformal weights of the massless states. We have worked out the number of generations, antigenerations and vectors for two depicted examples on the CFT side using our counting algorithm. On the geometry side we can compute the spectrum by the means of the χ-genus of DK models. Comparison of the spectra, in particular the number of generations and antigenerations, as obtained on the CFT and the geometry side show perfect agreement for a number of examples two of which have been depicted here. Since all examples we have considered have passed the test, there is strong evidence that the conjectured relation between heterotic $(0,2)$ Gepner-type and Disterl-Kachru models stands on solid grounds.
As an outlook, the tools we developed for the computation of the spectrum for this class of models can, in further work, be applied to models with smaller gauge groups like $E_4 \cong SU(5)$ or $E_3 \cong SU(3)\times SU(2)$. Furthermore, our construction might also lead to interesting implications in $(0,2)$ mirror symmetry, which is an interesting topic in its own right.

6 Workshop - “Strings at the LHC and in the Early Universe”

With my invitation to the KITP I was accepted to attend the workshop “Strings at the LHC and in the Early Universe” that took place over the duration of my stay. In various talks and discussions I had the great opportunity to learn more about the exciting and complex task of finding suitable models constructed from string theory, that can reproduce all the known features of particle physics observable at the energies that we can observe today, without producing “exotics“. String theory is probably the most promising candidate for a theory that unifies the strong and electroweak forces of the Standard Model and even incorporates gravity in the picture, hence providing a framework for quantum gravity. Its exploration over the past decades has led to a much better understanding about the structure of the theory and the mathematics behind. A lot of progress has been made but given the complexity of the theory - after all, its aim is to describe all fundamental forces, that are separated at low energies, within one single framework - there is still a lot to learn.

In this workshop a lot of different approaches to viable string models have been presented. Most of them are based on one of the five ten dimensional superstring theories (type I, type IIA, type IIB, heterotic $E_8 \times E_8$ and heterotic $SO(32)$), all of which are related by certain dualities and which are the constituents of an underlying theory eleven dimensional theory, called M-theory. The other models represented were constructed in the framework of F-theory, which is an auxiliary theory in twelve dimensions, that has revealed striking novelties about the properites of string theory. Moreover, F-theory and M-theory are, again, related by dualities, in particular F-theory is closely related to type IIB and heterotic string theory.

Apart from new insights into the very structure of string theory itself, promising models for particle physics as well as cosmology have been presented at the KITP. In the near future, the *Large Hadron Collider* (LHC) at CERN will test the physics beyond the Standard Model, which means that energies that have so far not yet been reached by human technologies will finally become accessible. In fact, the experiments currently running at CERN are very successful and the prospects for finding signatures that reveal new physics very soon is quite promising. In the workshop many propositions for testable models have been made along with explicit suggestions for which signatures to search in experiments.

Besides particle physics, also cosmological challenges have been presented during the cosmology focus week. The most famous and widely accepted model of how the Universe evolved is *inflation*, which claims to explain the flatness, homogeneity and isotropy of the observed Universe. According to inflationary theory, shortly after the *Big Bang* the Universe underwent a period of very rapid expansion, which allowed the quantum fluctuations to grow to cosmic size and provide the basis of the structure and life in the Universe we observe today. The inflationary epoch is the first part of the electroweak epoch that followed the Grand Unification epoch, where strong and electroweak forces were unified in a single framework of particle interaction. In recent decades critics on inflationary models have been posed and alternative models are being constructed, one of which is the *Big Bounce*, an oscillatory model where the first cosmological event in a Universe is not the Big Bang but the collaps, or *Big Crunch*, of a previous Universe. A great amount of cosmological models is constructed within the framework of string theory and provides parameters that can be tested in cosmological experiments. Currently, the Planck satellite, which has been launched in May 2009, is collecting data on the anisotropy in the cosmic microwave background radiation. As has been discussed quite enthusiastically between experimentalists and cosmologist during the workshop, the signatures Planck is going to reveal will put stringent constraints on viable cosmological models and some will simply be ruled out.

In particle physics and cosmology groups all over the world are currently carrying out experiments that provide novel data and, hence, novel insights into physics. We are, now, in the position to test, for the first time, theoretical models that go far beyond the Standard Model of particle physics and put stringent bounds on the Standard Model of cosmology. We are at the edge of confirming predictions of viable theories or, otherwise, ruling out models that are not realized in nature. The opportunity to attend this workshop, in which these topics have been discussed enthusiastically, was a unique experience for me and shaped my view about the current paradigms in theoretical physics.

7 Personal Experience

Personally, I have very much enjoyed my stay at the KITP in Santa Barbara. Never before, I have experienced a more fruitful environment for carrying out research. The institute provides a relaxed atmosphere for informal discussions on scientific topics, especially in the joint coffee breaks, where mutual exchange is preassigned. The UCSB is employing a considerable amount of very distinguished theoretical physicists which contributes to its renown. Due to the relaxed and informal atmosphere it is much easier for students to get to talk to the experts in their fields of interest. Moreover, since the KITP is host of many different programs year round - sometimes several at a time - the flow of ideas from different places all over the world to the lecture halls of the KITP is notable. I had the opportunity to talk to many different people and create new friendships and connections to students and postdocs all over the world. These connections not only enrich my personal life but also contribute to my academic network.

Outside the academic sphere, the location of the KITP and Santa Barbara is also quite advantageous. Situated at the pacific ocean just in front the Santa Barbara mountains, the KITP also provides a very healthy atmosphere to work and live in. The institute itself has its own shower rooms for visitors and staff and hence encourages a *mens sana in corpore sano.*

To wrap up, I have benefited a lot from the stay, for my research as well as personally, and I would recommend to anybody to take the great opportunity the Marshall Plan Foundation offers and stay some time in some renown US institution.

Appendix: $\mathcal{N} = 2$ SCA

The $\mathcal{N} = 2$ Superconformal Algebra is generated by the Laurent modes L_n, $G_r^{\pm}$, J_m of the energy-momentum tensor $T(z)$, its fermionic superpartners $G^{\pm}(z)$ that generate the $\mathcal{N} = 2$ supersymmetry, and a $U(1)$ current $J(z)$. In particular, states in the Hilbert space carry two labels, the conformal weight h and the $U(1)$ charge q, with respect to the zero modes of the Laurent modes $L_{n=0}$ and $J_{m=0}$, according to $L_0|h,q\rangle$ and $J_0|h,q\rangle = q|h,q\rangle$.

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