CHAPTER 4

A STACKELBERG OLIGOPOLY GENERAL EQUILIBRIUM MODEL

4.1 SETTING

4.1.1 INTRODUCTION

Assume two countries that are both rather small but not necessarily similar in size. There exist two industries in each country, sector 1 and sector 2. Both sectors face imperfect competition, two firms each play a non-cooperative game in quantity setting and set their quantities simultaneously (the Cournot case). Both industries - obviously not in the autarky case - deliver their goods to an integrated world market. Their only input is labor, which is internationally immobile but mobile between sectors and shows perfect competition on the factor market. This yields a uniform wage rate for each country. All goods of one sector are perfect substitutes, no significant product differentiation is possible. Firms pay a fixed cost for remaining in their industry, which also works as a sufficiently high barrier to entry for possible new incumbents. Returns to scale in production are assumed to be constant and there exist no transport cost for foreign consumption from world markets. Given this setting, it is obvious that the Ricardian cost structure should imply a partial or complete specialization in production in a general equilibrium solution as in (Samuelson 1964), depending on the voice of technology parameters. This feature is crucial in comparing the comparative cost advantages and implied welfare levels. The number of firms is neither endogenous nor will there be calculated a solution for n firms since this is not relevant to the findings. A fixed number of firms in a real oligopoly (contrary to monopolistic competition) has been the approach since (Wilcox 1950).

4.1.2 ORGANIZATION OF THE FIRM AND A SWITCH IN COMPETITION

I assume firms from the home industrial country (the home country h) are able to set up a distribution channel abroad. This is possible for the fixed cost of G_f.[21] Firms become multinationals by undergoing this horizontal FDI. But why should a firm open a distribution channel abroad if this does neither bypass transport cost nor decrease variable cost, increase demand for a certain differentiated product variation or decreases average cost through scale effects? The answer lies within λ, which will in detail be discussed in section 4.2.3 concerning its crucial link between demand and supply in a general equilibrium with a limited number of sectors. Lambda is the shadow price of income or the marginal utility of one unit of income for consumers.

It would be wrong to ignore time and assume a fully simultaneous general equilibrium without anything happening *before* the equilibrium is determined simultaneously in the given case with lots of strategic interaction through countries, factor incomes and firm behavior. Therefore, one can think of a certain period of time until firms gather all the information on λ, besides the schizophrenically but necessary fact that it is assumed to be fixed, as mentioned in the discussion in chapter 3 and 4.2.3. Again, λ has to be assumed fixed for firms in terms of wages and profits, because otherwise firms could generate demand through supply, a problem that has early been captured by the literature on imperfect competition in general equilibria, f.e. by (Negishi 1961). This *time window* between formulating the best response (the reaction functions) and the actual equilibrium output at given exogenous parameters depends also on λ, which itself contains the term Ω_i[22] and stands for the sum of all firms fixed cost. As calculated in section 4.2.2 in equation (4.9), lambda is nothing but:

$$\lambda^{consumer*} = \frac{a_1 b_2 p_1 + b_1 a_2 p_2 - b_1 b_2(\Pi_1 + \Pi_2 + \Lambda_i w) - b_1 b_2 \Omega}{b_1 p_2^2 + b_2 p_1^2}$$

Whatever the perceived demand is, the information on a_i, b_i and Λ_i are exogenous to the general equilibrium. The wages (w) are determined in the general equilibrium. Be-

[21] With no necessary condition whether $G_f <=> F_f <=> F_h$.

[22] Where i is: Autarky, Cournot or Stackelberg.

fore facing the general equilibrium generated wages, firms have to calculate with the experienced wages from information on the past or, through a second thinkable way, in a Schumpeterian fashion a *prix crié* on wages from trade union negotiations. The profits (Π_i) are unknown to everyone as well until the general equilibrium is solved. The remaining part is Ω, the sum of fixed cost from countries home and foreign. Now the point is: Without doubt it is easier and faster to obtain information on fixed cost at home. Therefore, if a firm is multinational and thereby has no more *foreign* competitors, it is faster in gathering this information. In other words:

Hypothesis 1. *A home firm can set its optimal quantity before the other competitors do if it is a multinational and therefore needs less time to collect information on foreign fixed cost in Ω, which is part of λ. The result is Stackelberg competition since best responses (reaction functions) have already been formulated by multinationals when national firms can do so.*

This is exactly what Stackelberg himself illustrates in a model, where on page 97 in the second paragraph of (von Stackelberg 1934) he declares: *"... der dargestellte Unterschied in der marktanalytischen Methode zwischen den beiden Anbietern bewirkt dieses asymmetrische Ergebnis."*[23] This difference in the organization of the firm, which can be found as an incentive to undergo FDI, f.e. in (Feenstra 2003), results in an information asymmetry concerning the *speed of obtaining information* rather than the sum of information a firm gathers when solving the general equilibrium (simultaneously). I will call this from now on the *time span of percolation.*

Table 4.1 shows the timeline and the idea of this timespan of percolation between formulating the optimal response in a general matter and finding the foreign values for the actual quantity. The industry leader (or as Stackelberg calls this to be a firm in the *independent position*) is able to answer before period ε elapses, while the followers (or dependent firms in the notion of (von Stackelberg 1934)) need this time to gather information on F_f if they are from h and F_h if they are from f. For simplicity it takes exactly the same amount of time (ε) in both directions. The information of the best response are known to all firms since they can observe foreign technology and wages,

[23] "... the displayed difference in the market analysis method between the suppliers leads to an asymmetric result."

the demand has no special foreign structure in λ except for the mentioned foreign fixed cost.

Table 4.1: Timeline of formulating the optimal quantity

	$\nearrow$	$\rightarrow$		optimal quantity non-leaders (Cournot)
best response by firms				
	$\searrow$	Quantity setting by leader	$\rightarrow$	Quantity setting by non-leaders (Stackelberg)
t		$]t; t+\varepsilon[$		$t+\varepsilon$

The strategic motivation will be further discussed in chapter 5 since this setting of imperfect competition and asymmetric speed of collecting information is perfectly in line with the idea of (von Stackelberg 1934) concerning the massive interaction on markets even in the absence of modern game theory and in the setting of a one shot simultaneous general equilibrium.

4.2 Demand

4.2.1 Linear quadratic preferences

The choice of preference settings is a crucial step in searching for a general oligopolistic equilibrium (GOLE). One of the best reviews of this key step can be found in (Neary 2003). The intention is not to find any arbitrary form of demand functions that simply *work* with oligopolies in general equilibrium but to find a formulation of demand that does not allow strategic complements on firm output, which would lead to Bertrand competition in a strategic game. Also, non-monotonicity of reaction functions is a not desired feature since this would lead to "jumps" in strategies depending on the share of firm output on a market. These jumps, that may be fully rational with respect to the profit function and fixed cost, make no sense though when looking at pure output in reaction functions as mentioned in (Neary 2007).

Section 4.2.2 will derive linear demand form quadratic preferences. This step may look trivial but is wisely chosen. If we assume a small number of industries, every oligopolist has to treat other industry product prices as given and constant. This approach, together with a constant perceived λ, as in (Negishi 1961) and (Marschak &

Selten 1974), yield the necessary feature of demand as the utility maximizing set of goods with limited budget for consumers: Linear, calculable and non-influenceable by firms and best choice for consumers.

4.2.2 FIRST ORDER CONDITIONS

Utility is assumed to be quadratic as mentioned in the previous section:

$$\begin{aligned} U &= u_1 + u_2, \\ &= a_1 x_1 - \frac{(b_1 x_1)^2}{2} + a_2 x_2 - \frac{(b_2 x_2)^2}{2}. \end{aligned} \tag{4.1}$$

Consumers generate income through three channels: labor by work, profits of imperfectly competing goods markets firms and sunk cost by paid fixed cost (Ω). These sunk cost, as well as profits from sector one (Π_1) and sector 2 (Π_2) are distributed in a lump sum fashion. Labor supply (Λ) is multiplied by the endogenous wage rate (w). Income is nothing but:

$$\begin{aligned} \Psi &= w_l \Lambda_l + w_f \Lambda_f + \Pi_1 + \Pi_2 + \Omega_i. \quad \text{with} \quad l = h, f, \\ &\text{and} \quad i = A, C, S \text{ for Autarky, Cournot, Stackelberg} \end{aligned} \tag{4.2}$$

The aggregated consumption part of the economy - preferences are assumed to be identical and homothetic - therefore maximizes the following Langrange:

$$\mathcal{L} = a_1 x_1 - \frac{(b_1 x_1)^2}{2} + a_2 x_2 - \frac{(b_2 x_2)^2}{2} + \lambda \left[\Psi - p_1 x_1 - p_2 x_2 \right] \tag{4.3}$$

Partial derivation to goods one and two and the shadow price of income are the three first order conditions (FOCs):

$$\frac{\partial \mathcal{L}}{\partial x_1} = a_1 - b_1 x_1 - \lambda p_1 = 0, \tag{4.4}$$

$$\frac{\partial \mathcal{L}}{\partial x_2} = a_2 - b_2 x_2 - \lambda p_2 = 0, \tag{4.5}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \Psi - p_1 x_1 - p_2 x_2 = 0. \tag{4.6}$$

Solving (4.4) to (4.6) yields:

$$x_1^{consumer*} = \frac{a_1p_2^2 - a_2p_1p_2 + b_2p_1(\Pi_1 + \Pi_2 + \Lambda w + \Omega)}{b_1p_2^2 + b_2p_1^2}, \tag{4.7}$$

$$x_2^{consumer*} = \frac{a_2p_1^2 - a_1p_1p_2 + p_2b_1(\Pi_1 + \Pi_2 + \Lambda w + \Omega)}{b_1p_2^2 + b_2p_1^2}, \tag{4.8}$$

$$\lambda^{consumer*} = \frac{a_1b_2p_1 + b_1a_2p_2 - b_1b_2(\Pi_1 + \Pi_2 + \Lambda w + \Omega)}{b_1p_2^2 + b_2p_1^2}. \tag{4.9}$$

Or, without forgetting that λ does not drop out and is equal to (4.9):

$$x_1^{consumer*} = \frac{a_1 - p_1\lambda}{b_1}, \tag{4.10}$$

$$x_2^{consumer*} = \frac{a_2 - p_2\lambda}{b_2}. \tag{4.11}$$

The following changes occur for the switch from autarky to trade between home country h and foreign country f: Λ_f is introduced (the foreign labor supply), profits are differentiated between by countries, the wage rate at home (w_h) and abroad (w_f) may differ, but since taste is identical among countries and there exists only one integrated world good market, the solution to the FOCs (4.4) to (4.6) does not change.

4.2.3 THE ROLE OF λ

Perceived demand for firms is linear since they treat λ as constant. This idea arises in (Negishi 1961) as the only solution to kinky demand curves. Only in a general equilibrium solution, the perceived demand curve by firms (which is now a partial equilibrium approach) and the real demand curves are equal (due to an equal value for λ), which means that this solution to profit maximization firms and utility maximization by consumers/workers is viable in the general equilibrium, or $\lambda_{real} = \lambda_{perceived}$. In the further discussion on supply, the shadow price of income λ will be perceived by firms, especially due to its properties on foreign fixed cost and the time lag mentioned in section 4.1.2. The sections up to 4.2.3 have talked about real lambda since this was a utility maximization by consumers/workers. The sections on general equilibrium solutions need no further discrimination between real and perceived since those two have to be equal to obtain a general equilibrium in the system. The demand system and its properties, that have been discussed in section 4.2.1 are necessary to guarantee the mentioned substitutability of output of firms and allow a linear and thus monotonic reaction function in λ.

4.3 SUPPLY

4.3.1 PROFIT MAXIMIZATION

4.3.1.1 Autarky

In autarky, two firms in both of the two markets compete against each other. The assumption is that all markets face imperfect competition to a certain degree in order not to allow the wage - which equalizes between sectors but later on cannot do so internationally due to the immobility of labor - to be formulated at a perfectly competing (bigger) industry, which would somehow weaker the general equilibrium. Firms face the standard profit function:

$$\pi_{ji} = (p_j - c_j)x_{ji} - F \quad \text{with} \quad j = 1,2 \quad \text{is the sector and} \tag{4.12}$$

$$i = 1,2 \quad \text{is the firm.} \tag{4.13}$$

Firms are equal in productivity and products are perfect substitutes. In the autarky case it is obvious that $\Omega_A = 4F$. Fixed cost are assumed to be sufficiently high to prevent other entrepreneurs from entering the industries. Marginal cost are constant:

$$c_j = \alpha_j w_l. \tag{4.14}$$

The technology coefficient α_j is exogenous, the wage w_l for $l = h, f$ endogenous and both may differ between countries ($\alpha_{hj} \neq \alpha_{fj}$ and $w_h \neq w_f$). Firms know from the consumer utility maximization that direct demand is $x_j^* = \frac{a_j - p_j\lambda}{b_j}$, but they *do not know* that they could influence demand by raising supply and thereby paying more wages and profits to workers and shareholders respectively, i.e. supply must not create its own demand. This assumption could also have been made the other way round, but I assume it to be more realistic to maximize firm profits with given demand rather than maximize utility and avoid consumption financed over higher supply as an effect of higher demand and therefore income. This discussion about a necessary schizophrenia can also be found in f.e. (Ruffin 2003b). Deriving the firm FOCs in autarky is nothing but maximizing profits at the same time and mutual implementation of the opponents reaction function (Cournot):

$$\frac{\partial \pi_{ji}}{\partial x_{ji}} = (p_j - c_j) - \frac{\partial p_j}{\partial x_{ji}} x_{ji} \quad \text{for} \quad j = 1,2;\ i = 1,2. \tag{4.15}$$

This yields the reaction functions:

$$x_{ji}^{reaction} = \frac{a_j - b_j x_{ki} - w\alpha_j \lambda}{2b_j} \quad \text{for} \quad j,k = 1,2;\ j \neq k;\ i = 1,2. \tag{4.16}$$

Simultaneous quantity setting (the Cournot case), therefore yields optimal (Nash equilibrium) quantities:

$$x_{ji}^{firm*} = \frac{a_j - w\alpha_j \lambda}{3b_j}, \tag{4.17}$$

$$x_j^{sector*} = \frac{2(a_j - w\alpha_j \lambda)}{3b_j} \quad \text{for} \quad j = 1,2. \tag{4.18}$$

Knowing this one can easily calculate the optimal price and profit:

$$p_j^{firm*} = \frac{a_j - b_j \frac{2(a_j - w\alpha_j\lambda)}{3b_j}}{\lambda}, \tag{4.19}$$

$$\pi_{ji}^{firm*} = \left(\frac{a_j - b_j x_j}{\lambda} - \alpha_j w\right) x_{ji} - F \quad \text{for} \quad j = 1,2;\ i = 1,2. \tag{4.20}$$

Ω in this case is nothing but the sum of fixed cost F paid by two firms in two sectors or $\Omega_A \equiv 4F$.

4.3.1.2 *Trade and Cournot competition*

The four firms per sector, two in each country face the following profit function:

$$\pi_{h11} = (p_1 - c_{h1})x_{h11} - F_h, \tag{4.21}$$

$$\pi_{h12} = (p_1 - c_{h1})x_{h12} - F_h, \tag{4.22}$$

$$\pi_{h21} = (p_2 - c_{h2})x_{h21} - F_h, \tag{4.23}$$

$$\pi_{h22} = (p_2 - c_{h2})x_{h22} - F_h, \tag{4.24}$$

$$\pi_{f11} = (p_1 - c_{f1})x_{f11} - F_f, \tag{4.25}$$

$$\pi_{f12} = (p_1 - c_{f1})x_{f12} - F_f, \tag{4.26}$$

$$\pi_{f21} = (p_2 - c_{f2})x_{f21} - F_f, \tag{4.27}$$

$$\pi_{f22} = (p_2 - c_{f2})x_{f22} - F_f. \tag{4.28}$$

Now, the fixed cost are higher, or $\Omega_C \equiv 4F_h + 4F_f$. Each firm simultaneously determines the quantities by maximizing profits with respect to x_{lji} for country l, sector j and firm i, or:

$$\frac{\partial \pi_{lji}}{\partial x_{lji}} = (p_j - c_{lj}) + \frac{\partial p_j}{\partial x_{lji}} x_{lji} \quad \text{for} \quad l = h, f;\ j = 1, 2;\ i = 1, 2. \tag{4.29}$$

(4.29) are the FOCs of all eight firms and leads to the reaction functions:

$$x_{lji}^{reaction} = \frac{a_j - w_l \alpha_{lj} \lambda - b_j(x_{ljk} + x_{mji} + x_{mjk})}{2b_j} \tag{4.30}$$

$$\text{for} \quad l, m = h, f;\ l \neq m;\ j = 1, 2;\ i, k = 1, 2;\ i \neq k.$$

The reaction functions from (4.44), given with the fact that both firms of a certain country and sector will set identical quantities allows to solve the set of reaction functions to obtain the eight profit maximizing quantitites:

$$x_{lji}^{firm*} = \frac{a_j + 2w_m \alpha_{mj} \lambda - 3w_l \alpha_{lj} \lambda}{5b_j} \tag{4.31}$$

$$\text{for} \quad l, m = h, f;\ l \neq m;\ j = 1, 2;\ i = 1, 2.$$

4.3.1.3 *Trade and Stackelberg competition with one leader*

Starting at equations (4.29) and (4.44), where firms formulate their best response to every setting of demand, technology and income to their competitors, the move order changes. Assume that in sector two - instead of simultaneous quantity setting - firm one of sector two maximizes its quantity (x_{h21}) before the others do because it has decided to undergo the FDI G_f and has become a multinational. From rational best responses it knows its competitors reactions functions:

$$\frac{\partial \pi_{h22}}{\partial x_{h22}} = 0 = \frac{a_2 - b_2\left(x_{f21} + x_{f22} + x_{h21} + x_{h22}\right)}{\lambda} - \frac{b_2 x_{h22}}{\lambda} - \alpha_{h2} w_h, \quad (4.32)$$

$$\frac{\partial \pi_{f21}}{\partial x_{f21}} = 0 = \frac{a_2 - b_2\left(x_{f21} + x_{f22} + x_{h21} + x_{h22}\right)}{\lambda} - \frac{b_2 x_{f21}}{\lambda} - \alpha_{f2} w_f, \quad (4.33)$$

$$\frac{\partial \pi_{f22}}{\partial x_{f22}} = 0 = \frac{a_2 - b_2\left(x_{f21} + x_{f22} + x_{h21} + x_{h22}\right)}{\lambda} - \frac{b_2 x_{f22}}{\lambda} - \alpha_{f2} w_f. \quad (4.34)$$

The multinational firm can now maximize its profits π_{h21}^{leader} knowing the competitors reaction from (4.32) to (4.34). To do so, the firm first has to eliminate the reaction of its opponents to the other two opponents (i.e. eliminate three unknows out of three equations):

$$x_{h22}^{reaction} = \frac{a_2 - b_2 x_{h21}^{leader} + 2 w_f \alpha_{f2} \lambda - 3 w_h \alpha_{h2} \lambda}{4 b_2}, \quad (4.35)$$

$$x_{f21}^{reaction} = \frac{a_2 - b_2 x_{h21}^{leader} - 2 w_f \alpha_{f2} \lambda + w_h \alpha_{h2} \lambda}{4 b_2}, \quad (4.36)$$

$$x_{f22}^{reaction} = \frac{a_2 - b_2 x_{h21}^{leader} - 2 w_f \alpha_{f2} \lambda + w_h \alpha_{h2} \lambda}{4 b_2}. \quad (4.37)$$

Equations (4.35) to (4.37) are now being inserted in the profit function of the leader *before* this firms derives its optimal quantity. By doing so it the leader takes into account the optimal reaction of the followers to its decision, which follows later as stage two. Thus, step one is nothing but:

$$\pi_{h21}^{leader*} = \left(p_2^{leader} - c_2\right) x_{h21}^{leader} - F_h - G_f, \tag{4.38}$$

$$= \left(\frac{a_2 - b_2\left(x_2^{leader}\right)}{\lambda} - \alpha_{h2} w_h\right) x_{h21}^{leader} - F_h - G_f,$$

$$\text{with } x_2^{leader} \equiv x_{h22}^{reaction} + x_{f21}^{reaction} + x_{f22}^{reaction} + x_{h21}^{leader},$$

$$\text{or } x_2^{leader} = \frac{3a_2 + b_2 x_{h21} - 2w_f \alpha_{f2}\lambda - w_h \alpha_{h2}\lambda}{4b_2},$$

$$\frac{\partial \pi_{h21}^{leader}}{\partial x_{h21}^{leader}} = 0 \;\Leftrightarrow x_{h21}^{leader*} = \frac{a_2 + 2w_f \alpha_{f2}\lambda - 3w_h \alpha_{h2}\lambda}{2b_2}. \tag{4.39}$$

Step two is the answer by the three followers, one from country h and two from f. They can not afford to veer their best response and react according to their reaction functions (4.35) to (4.37) respectively:

$$x_{h22}^{follower*} = \frac{a_2 + 2w_f \alpha_{f2}\lambda - 3w_h \alpha_{h2}\lambda}{8b_2}, \tag{4.40}$$

$$x_{f21}^{follower*} = \frac{a_2 - 6w_f \alpha_{f2}\lambda + 5w_h \alpha_{h2}\lambda}{8b_2}, \tag{4.41}$$

$$x_{f22}^{follower*} = \frac{a_2 - 6w_f \alpha_{f2}\lambda + 5w_h \alpha_{h2}\lambda}{8b_2}. \tag{4.42}$$

Equations (4.40) to (4.42) show that the followers in the countries do not react with the same strength to a variation in the comparative cost structure. A detailed analysis of price-making and leader power versus comparative cost advantages follows in section 5.4. Sector one is still in Cournot competition as in section 4.3.1.2, therefore the optimal quantity remains unchanged (from a partial equilibrium standpoint of supply - the general equilibrium will reveal a change in profits and wages and therefore a different demand through income and λ for all sectors):

$$x_{l1i}^{firm*} = \frac{a_1 + 2w_m \alpha_{m1}\lambda - 3w_l \alpha_{l1}\lambda}{5b_1} \quad \text{for } l, m = h, f;\; l \neq m;\; i = 1, 2. \tag{4.43}$$

4.3.1.4 Trade and Stackelberg competition with two leaders

This is just a slight variation to the previous section. Now, both home firms of sector two undergo the foreign direct investment G_f to speed up in gathering information on λ and thus being able to formulate their optimal quantities earlier. In other words, both home firms set their quantities simultaneously and before the foreign firms set their quantities simultaneously. Best responses have not changed and are still:

$$
\begin{aligned}
x_{lji}^{reaction} &= \frac{a_j - w_l \alpha_{lj}\lambda - b_j(x_{ljk} + x_{mji} + x_{mjk})}{2b_j} \\
& \text{for} \quad l, m = h, f;\ l \neq m;\ j = 1, 2;\ i, k = 1, 2;\ i \neq k.
\end{aligned}
$$

Step one is profit maximization of home firms, which both know about this reaction by foreign firms. Deleting the information on the reaction on the other foreign firms leads to:

$$
x_{f21}^{reaction} = \frac{a_2 - b_2 x_{h21}^{leader} - b_2 x_{h22}^{leader} - w_f \alpha_{f2}\lambda}{3b_2}, \tag{4.44}
$$

$$
x_{f22}^{reaction} = \frac{a_2 - b_2 x_{h21}^{leader} - b_2 x_{h22}^{leader} - w_f \alpha_{f2}\lambda}{3b_2}. \tag{4.45}
$$

Home firms therefore face the following profit maximization problem:

$$
\begin{aligned}
\pi_{h2i} &= \left(p_2^{2leader} - c_2\right) x_{h2i}^{2leader} - F_h - G_f, \quad \text{with} \quad i = 1, 2, \qquad (4.46) \\
&= \left(\frac{a_2 - b_2\left(x_2^{2leader}\right)}{\lambda} - \alpha_{h2} w_h \right) x_{h2i}^{2leader} - F_h - G_f, \\
& \text{with } x_2^{2leader} \equiv x_{h21}^{2leader} + x_{h22}^{2leader} + x_{f21}^{reaction} + x_{f22}^{reaction}, \\
& \text{or } x_2^{2leader} = \frac{2a_2 + b_2(x_{h21} + x_{h22}) - 2w_f \alpha_{f2}\lambda}{3b_2},
\end{aligned}
$$

$$
\frac{\partial \pi_{h2i}^{2leader}}{\partial x_{h2i}^{2leader}} = 0 \Leftrightarrow x_{h2i}^{2leader*} = \frac{a_2 + 2w_f \alpha_{f2}\lambda - 3w_h \alpha_{h2}\lambda}{3b_2}. \tag{4.47}
$$

Step two is profit maximization of foreign firms after watching the chosen quantities of their industry leaders and competitors from the country h:

$$x_{f21}^{follower*} = \frac{a_2 - 7w_f\alpha_{f2}\lambda + 6w_h\alpha_{h2}\lambda}{9b_2}, \tag{4.48}$$

$$x_{f22}^{follower*} = \frac{a_2 - 7w_f\alpha_{f2}\lambda + 6w_h\alpha_{h2}\lambda}{9b_2}. \tag{4.49}$$

Of course, in this setting one cannot see a difference in the strength of reaction among followers since both are located in country f, although a reaction to a change in the cost of country f hits the followers harder (by factor 7) than a change in the cost of country h (by factor 6). Again, the resulting implications and thresholds are the topic of section 5.4. The quantities of sector one remain unchanged from the Cournot case and are still:

$$x_{l1i}^{firm*} = \frac{a_1 + 2w_m\alpha_{m1}\lambda - 3w_l\alpha_{l1}\lambda}{5b_1} \quad \text{for } l,m = h,f;\ l \neq m;\ i = 1,2.$$

4.3.2 FACTOR MARKETS

The only input needed in production is uniform labor. National endowments of labor in home h and foreign f can be different and are Λ_h and Λ_f respectively. Full employment and a competitive factor market are assumed, since the low number of firms at home and abroad lead to imperfect competition on product markets but a lot of workers at home and abroad lead to perfect competition on the factor market. Full employment is a common simplification assumption that will not lead to distorted results. At the time firms choose their quantities to perceived demand they obtain information about the amount of labor they need as an input. Given the cost functions are $c_{lj} = \alpha_l w_l$ with $j = 1,2$ as in equation (4.14) but now of course also with $l = h, f$ for different technologies and wage rates in both countries, demand for labor is nothing but:

$$l_{lj}^{demand} = \alpha_{lj}x_j^* \quad \text{with} \quad l = h,f;\ j = 1,2. \tag{4.50}$$

Given the fact that labor is perfectly mobile between industries but not internationally leads to a nationally uniform wage rate. Market clearing for the factor market thus is nothing but supply through endowment equals demand arising by profit maximization, or:

$$\Lambda_h \equiv l_{h1} + l_{h2} = \alpha_{h1} x^*_{h1} + \alpha_{h2} x^*_{h2} \quad \text{for the home country } h \text{ and} \tag{4.51}$$

$$\Lambda_f \equiv l_{f1} + l_{f2} = \alpha_{f1} x^*_{f1} + \alpha_{f2} x^*_{f2} \quad \text{for the foreign country } f. \tag{4.52}$$

4.4 EQUILIBRIUM IN AUTARKY

4.4.1 CHOOSING A NUMÉRAIRE

While there must not be any difference in the results of a general equilibrium with perfect competition if the numeraire is changed, there are serious concerns that the choice of the numeraire has a major impact on the solution under oligopoly or monopoly as pointed out by (Cornwall 1977), (Böhm 1994) and (Willenbockel 2004). The assumption of a continuum of sectors bypasses this problem by stating that a firm is large in its sector but infinitesimal small on an economy wide level. This has been the approach by Neary in (Neary 2003) and (Neary 2007). For a small number of sectors such as in (Gabszewicz & Vial 1972) and (Ruffin 2003*b*), the assumption of a schizophrenically acting firm is made.[24] Both approach share a common feature: The bypasses avoid massive problems in the choice of the numeraire, else - as (Willenbockel 2004) shows - not doing so would allow to make every desired output level (or price) the equilibrium quantity and price. (Ruffin 2003*b*) calls this problem a *red herring*, hinting at the fact that choosing a certain numéraire makes no or perfect sense, depending not on the choice but the problem of limiting *general* equilibrium by introduction perceived (i.e not open to influence) demand, scientific theory would call this an ignoratio elenchi. (Gabszewicz & Vial 1972) mention that *rejecting a choice of the numéraire would imply rejecting imperfect competition in partial equilibrium as a whole.*

The approach of this dissertation is to assume firms to be schizophrenically and to differ between real and perceived demand - as discussed in other chapters. It is the designated feature of an incomplete market that firms can influence prices and thereby income in a general equilibrium. A very high number of sectors, a non-traded big part of the economy or the introduction of money as a neutral valve through expectations

[24]One could also state this problem as implementing partial equilibrium *ceteris paribus analysis* into a general equilibrium framework.

are just different ways to bypass the very same problem. Since the focus of this work is not to set up a new approach on this problem but research on the motivation for market leadership, a possible explanation for multinational activity through features in the organization of the firm and the trade-off between leadership status and comparative cost structures and endowments, I will set the price of good one $p_1 = 1$, given the assumption that firms in this model do not realize their possible influence on the whole economy, it would make no difference to choose any other endogenous variable for the numéraire. Moreover, the approach presented by (Negishi 1961) is one that may sound artificial. Though, to discriminate between real and perceived demand in a model with - for the sake of simplicity - two sectors is just a bypass to model a real economy with thousands of (but not indefinitely many) sectors as long as one can bear the fact that assumptions may lead to minor deviations. Literature and research has shown so far that these deviations are very likely small enough not to invalidate the presented findings on strategic interaction, welfare and trade patterns.

4.4.2 SOLUTION

The equation system in autarky is well-defined by the following system in table 4.2. The twelve endogenous variables are $\Psi, p_1, p_2, x_1, x_2, l_1, l_2, w, \lambda, \Pi_1, \Pi_2, \Pi$, of which p_1, the price of good one is the numéraire, or $p_1 = 1$. This leaves eleven equations for eleven endogenous variables. The solution to the system of table 4.2 will be presented in the text, the general solutions to the trade in Stackelberg and Cournot competition cases will be found in detail in the appendix.[25] The general solution - the case without any further information on the exogenous variables $\Lambda, \alpha_1, \alpha_2$ and F - thus is written in equations (4.53) to (4.64), mind the slightly different notation in the indexes since this is Mathematica output. The SOLVE[] loop of Mathematica uses a *Gaussian elimination algorithm*. It can eliminate n endogenous variables from n independent equations using $2n^3/3$ equations. Thus, computational assistance in solving these models even though they have 20-30 endogenous variables becomes complex to the power of three. The Gaussian elimination is a simple but efficient algorithm. It rearranges equation by equation to eliminate one variable at a time, iterating this procedure for every single endogenous variable until no endogenous variable is left in the solution. If the pro-

[25]The solution to the autarky case is short enough to present it representatively for all general cases.

Table 4.2: Autarky with Cournot competition

2 product markets	
demand	$x_1 = \frac{a_1 - p_1\lambda}{b_1}$
	$x_2 = \frac{a_2 - p_2\lambda}{b_2}$
supply	$x_1 = \frac{2(a_1 - w\alpha_1\lambda)}{3b_1}$
	$x_2 = \frac{2(a_2 - w\alpha_2\lambda)}{3b_2}$
	$x_{11} = x_{12} = \frac{x_1}{2}$
	$x_{21} = x_{22} = \frac{x_2}{2}$

factor market	
demand	$l_1 = \alpha_1 w$
	$l_2 = \alpha_2 w$
supply	Λ_h
market clearing	$\Lambda_h = l_1 + l_2$

income equations	
consumers	$\Phi = wL + \Pi + \Omega$
	$\Omega = 4F$
firms	$\Pi = \Pi_1 + \Pi_2$
	$\Pi_1 = (p_1 - c_1)x_1 - 2F$
	$\Pi_2 = (p_1 - c_2)x_2 - 2F$
	$c_1 = \alpha_1 w$
	$c_2 = \alpha_2 w$

cedure is applied on a linear system as the one presented in table 4.53 (and also the other upcoming general equilibria), there is only one single equilibrium solution possible, which eases economic determination and interpretation tremendously. It does though not tell us if the outcome makes perfect economic or sense, or if f.e. firm profits are negative, which would be a non-feasible outcome. This reduction algorithm can be found long before the tremendous oeuvre of the German mathematician and physicist Carl Friedrich Gauß (1777-1855). The Chinese *nine books of arithmetic technique*, written between 200 BC and 100 AD, have already implemented this elimination method. European scientists have not used it explicitly until Lagrange and Gauß. More complex problems that reach the constraints of modern computing need iteration methods (ways of splitting the endogenous variable matrices), but the models presented in this work are computed in less than one minute.

$$\mathrm{p1} \rightarrow 1, \tag{4.53}$$

$$\mathrm{p2} \rightarrow \frac{2\mathrm{a1}\alpha\mathrm{1}\alpha\mathrm{2b2} + 3\mathrm{a2}\alpha\mathrm{2}^2\mathrm{b1} + \alpha\mathrm{1}^2\mathrm{a2b2} - 3\alpha\mathrm{2b1b2}\Lambda\mathrm{h}}{\mathrm{a1}\alpha\mathrm{2}^2\mathrm{b1} + 3\mathrm{a1}\alpha\mathrm{1}^2\mathrm{b2} + 2\alpha\mathrm{1a2}\alpha\mathrm{2b1} - 3\alpha\mathrm{1b1b2}\Lambda\mathrm{h}}, \tag{4.54}$$

$$\begin{aligned}\Psi \rightarrow\ & \frac{2\alpha\mathrm{1}^2\left(18\mathrm{a1b2}F + \mathrm{a2}^2\right) + 9\Lambda\mathrm{h}(\alpha\mathrm{1b2}(\mathrm{a1} - 4\mathrm{b1}F) + \mathrm{a2}\alpha\mathrm{2b1})}{3\mathrm{a1}\alpha\mathrm{2}^2\mathrm{b1} + 9\mathrm{a1}\alpha\mathrm{1}^2\mathrm{b2} + 6\alpha\mathrm{1a2}\alpha\mathrm{2b1} - 9\alpha\mathrm{1b1b2}\Lambda\mathrm{h}} \\ & - \frac{4\alpha\mathrm{1a2}\alpha\mathrm{2}(\mathrm{a1} - 6\mathrm{b1}F) + 2\mathrm{a1}\alpha\mathrm{2}^2(\mathrm{a1} + 6\mathrm{b1}F) - 9\mathrm{b1b2}\Lambda\mathrm{h}^2}{3\mathrm{a1}\alpha\mathrm{2}^2\mathrm{b1} + 9\mathrm{a1}\alpha\mathrm{1}^2\mathrm{b2} + 6\alpha\mathrm{1a2}\alpha\mathrm{2b1} - 9\alpha\mathrm{1b1b2}\Lambda\mathrm{h}},\end{aligned} \tag{4.55}$$

$$\mathrm{x1} \rightarrow \frac{2\mathrm{a1}\alpha\mathrm{2}^2 - 2\alpha\mathrm{1a2}\alpha\mathrm{2} + 3\alpha\mathrm{1b2}\Lambda\mathrm{h}}{3\left(\alpha\mathrm{2}^2\mathrm{b1} + \alpha\mathrm{1}^2\mathrm{b2}\right)}, \tag{4.56}$$

$$\mathrm{x2} \rightarrow \frac{-2\mathrm{a1}\alpha\mathrm{1}\alpha\mathrm{2} + 2\alpha\mathrm{1}^2\mathrm{a2} + 3\alpha\mathrm{2b1}\Lambda\mathrm{h}}{3\alpha\mathrm{2}^2\mathrm{b1} + 3\alpha\mathrm{1}^2\mathrm{b2}}, \tag{4.57}$$

$$\mathrm{l1} \rightarrow \frac{\alpha\mathrm{1}\left(2\mathrm{a1}\alpha\mathrm{2}^2 - 2\alpha\mathrm{1a2}\alpha\mathrm{2} + 3\alpha\mathrm{1b2}\Lambda\mathrm{h}\right)}{3\left(\alpha\mathrm{2}^2\mathrm{b1} + \alpha\mathrm{1}^2\mathrm{b2}\right)}, \tag{4.58}$$

$$\mathrm{l2} \rightarrow \frac{\alpha\mathrm{2}\left(-2\mathrm{a1}\alpha\mathrm{1}\alpha\mathrm{2} + 2\alpha\mathrm{1}^2\mathrm{a2} + 3\alpha\mathrm{2b1}\Lambda\mathrm{h}\right)}{3\left(\alpha\mathrm{2}^2\mathrm{b1} + \alpha\mathrm{1}^2\mathrm{b2}\right)}, \tag{4.59}$$

$$w \rightarrow \frac{6\mathrm{a1}\alpha\mathrm{1b2} + 6\mathrm{a2}\alpha\mathrm{2b1} - 9\mathrm{b1b2}\Lambda\mathrm{h}}{2\mathrm{a1}\alpha\mathrm{2}^2\mathrm{b1} + 6\mathrm{a1}\alpha\mathrm{1}^2\mathrm{b2} + 4\alpha\mathrm{1a2}\alpha\mathrm{2b1} - 6\alpha\mathrm{1b1b2}\Lambda\mathrm{h}}, \tag{4.60}$$

$$\lambda \rightarrow \frac{\mathrm{a1}\alpha\mathrm{2}^2\mathrm{b1} + 3\mathrm{a1}\alpha\mathrm{1}^2\mathrm{b2} + 2\alpha\mathrm{1a2}\alpha\mathrm{2b1} - 3\alpha\mathrm{1b1b2}\Lambda\mathrm{h}}{3\alpha\mathrm{2}^2\mathrm{b1} + 3\alpha\mathrm{1}^2\mathrm{b2}}, \tag{4.61}$$

$$\begin{aligned}\Pi\mathrm{1} \rightarrow\ & \frac{(4\mathrm{a1}^2\alpha\mathrm{2}^4\mathrm{b1} - 4\mathrm{a1}\left(\alpha\mathrm{2}^3\mathrm{b1}(2\alpha\mathrm{1a2} + 3\alpha\mathrm{2b1}F) + 3\alpha\mathrm{1}\alpha\mathrm{2}^2\mathrm{b1b2}(4\alpha\mathrm{1}F - \Lambda\mathrm{h}) + 9\alpha\mathrm{1}^4\mathrm{b2}^2F\right)}{\left(6\left(\alpha\mathrm{2}^2\mathrm{b1} + \alpha\mathrm{1}^2\mathrm{b2}\right)\left(\mathrm{a1}\alpha\mathrm{2}^2\mathrm{b1} + 3\mathrm{a1}\alpha\mathrm{1}^2\mathrm{b2} + 2\alpha\mathrm{1a2}\alpha\mathrm{2b1} - 3\alpha\mathrm{1b1b2}\Lambda\mathrm{h}\right)\right)} \\ & + \frac{\alpha\mathrm{1b1}(2\mathrm{a2}\alpha\mathrm{2} - 3\mathrm{b2}\Lambda\mathrm{h})(2\alpha\mathrm{2}(\alpha\mathrm{1a2} - 6\alpha\mathrm{2b1}F) - 3\alpha\mathrm{1b2}(4\alpha\mathrm{1}F + \Lambda\mathrm{h}))}{\left(6\left(\alpha\mathrm{2}^2\mathrm{b1} + \alpha\mathrm{1}^2\mathrm{b2}\right)\left(\mathrm{a1}\alpha\mathrm{2}^2\mathrm{b1} + 3\mathrm{a1}\alpha\mathrm{1}^2\mathrm{b2} + 2\alpha\mathrm{1a2}\alpha\mathrm{2b1} - 3\alpha\mathrm{1b1b2}\Lambda\mathrm{h}\right)\right)},\end{aligned} \tag{4.62}$$

$$\begin{aligned}\Pi\mathrm{2} \rightarrow\ & \frac{4\left(\alpha\mathrm{1}^4\mathrm{b2}\left(\mathrm{a2}^2 - 9\mathrm{a1b2}F\right) - 2\alpha\mathrm{1}^3\mathrm{a2}\alpha\mathrm{2b2}(\mathrm{a1} + 3\mathrm{b1}F) - 3\mathrm{a1}\alpha\mathrm{2}^4\mathrm{b1}^2F + \mathrm{a1}\alpha\mathrm{1}^2\alpha\mathrm{2}^2\mathrm{b2}(\mathrm{a1} - 12\mathrm{b1}F) - 6\alpha\mathrm{1a2}\alpha\mathrm{2}^3\mathrm{b1}^2F\right)}{6\left(\alpha\mathrm{2}^2\mathrm{b1} + \alpha\mathrm{1}^2\mathrm{b2}\right)\left(\mathrm{a1}\alpha\mathrm{2}^2\mathrm{b1} + 3\mathrm{a1}\alpha\mathrm{1}^2\mathrm{b2} + 2\alpha\mathrm{1a2}\alpha\mathrm{2b1} - 3\alpha\mathrm{1b1b2}\Lambda\mathrm{h}\right)} \\ & + \frac{12\alpha\mathrm{1b1b2}\Lambda\mathrm{h}\left(\alpha\mathrm{2}(-\mathrm{a1}\alpha\mathrm{2} + \alpha\mathrm{1a2} + 3\alpha\mathrm{2b1}F) + 3\alpha\mathrm{1}^2\mathrm{b2}F\right) + 9\alpha\mathrm{2}^2\mathrm{b1}^2\mathrm{b2}\Lambda\mathrm{h}^2}{6\left(\alpha\mathrm{2}^2\mathrm{b1} + \alpha\mathrm{1}^2\mathrm{b2}\right)\left(\mathrm{a1}\alpha\mathrm{2}^2\mathrm{b1} + 3\mathrm{a1}\alpha\mathrm{1}^2\mathrm{b2} + 2\alpha\mathrm{1a2}\alpha\mathrm{2b1} - 3\alpha\mathrm{1b1b2}\Lambda\mathrm{h}\right)},\end{aligned} \tag{4.63}$$

$$\Pi \rightarrow \frac{4\mathrm{a1}^2\alpha\mathrm{2}^2 - 8\alpha\mathrm{1a2}\alpha\mathrm{2}(\mathrm{a1} + 6\mathrm{b1}F) - 24\mathrm{a1}F\left(\alpha\mathrm{2}^2\mathrm{b1} + 3\alpha\mathrm{1}^2\mathrm{b2}\right) + 4\alpha\mathrm{1}^2\mathrm{a2}^2 + 9\mathrm{b1b2}\Lambda\mathrm{h}(8\alpha\mathrm{1}F + \Lambda\mathrm{h}}{6\left(\mathrm{a1}\alpha\mathrm{2}^2\mathrm{b1} + 3\mathrm{a1}\alpha\mathrm{1}^2\mathrm{b2} + 2\alpha\mathrm{1a2}\alpha\mathrm{2b1} - 3\alpha\mathrm{1b1b2}\Lambda\mathrm{h}\right.}. \tag{4.64}$$

This general solution leaves of course no room for a further understandable or straight forward analysis of the result. To do so, it is tremendously helpful to add numerical values for the exogenous variables. They are $a_1 = a_2 = 80$, $b_1 = b_2 = 10$ and

$\Lambda_h = 100$. This yields f.e.:

$$x_1 = \frac{4\left(\alpha_1(75 - 4\alpha_2) + 4{\alpha_2}^2\right)}{3\left({\alpha_1}^2 + {\alpha_2}^2\right)},$$

$$x_2 = \frac{4\left(4{\alpha_1}^2 + 75\alpha_2 - 4\alpha_1\alpha_2\right)}{3\left({\alpha_1}^2 + {\alpha_2}^2\right)}.$$

Fixing $\alpha_2 = 30$ allows to plot the following graph: Figure 4.1 shows that as long as

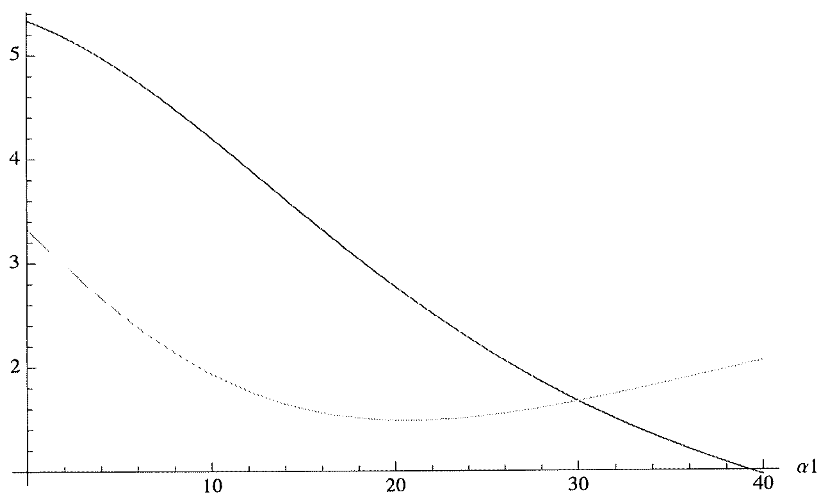

Figure 4.1: Volume of production of good x_1 (red) and x_2 (green) at $\alpha_2 = 30$.

production of good one is relatively cheaper and both goods are assumed to deliver the same utility ($a_1 = a_2$ and $b_1 = b_2$ - no strict preference on the equal amount of goods) the output of good x_1 will be higher. At $\alpha_1 = 30 = \alpha_2$ both goods will be produced (and thereby consumed) in equal quantities and the production of good x_2 will overtake as soon as good x_1 gets more expensive in production. Note that figure 4.1 already shows a very important feature of the latter trade general oligopolistic equilibria: Above $\alpha_1 = 40$, the equilibrium production of good x_1 becomes negative, which makes no economic sense. In other words: Not any arbitrary combination of exogenous factor values delivers a general equilibrium, or more precisely: Relative production cost must not be too extreme. To display a possible outcome, the following values describe a general equilbrium: Table 4.3 makes perfectly economic sense. The

Table 4.3: General oligopolistic equilibrium in autarky at $\alpha_1 = 31$, $\alpha_2 = 30$, $a_1 = a_2 = 80$, $b_1 = b_2 = 10$ and $\Lambda_h = 100$

$$\begin{pmatrix} \Phi \to 3.248595767 & p_1 \to 1 & p_2 \to 0.981140498 \\ x_1 \to 1.579795809 & x_2 \to 1.700877664 & l_1 \to 48.97367007 \\ l_2 \to 51.02632993 & w \to 0.028289253 & \lambda \to 64.202041913 \\ \Pi_1 \to 0.194367245 & \Pi_2 \to 0.225303179 & \Pi \to 0.419670424 \end{pmatrix}$$

(equilibrium) price of good two p_2 is relatively small compared to good one since at indifference between the two goods in the consumers preferences and a slightly lower cost of production ($\alpha_2 = 30 < \alpha_1 = 31$) marginal revenue cross marginal cost at a lower price in good x_2. The amount produced is thus higher in good x_2, which also means that the amount of labor spent for production of good two is slightly larger ($l_2 > l_1$) and, of course, $l_2 + l_1 = 100 = \Lambda_h$. At equal preferences for good one and two, lower cost of production and a higher production, firms in sector 2 make more profits ($\Pi_1 < \Pi_2$) and again, $\Pi_1 + \Pi_2 = \Pi$.

4.5 Equilibrium in Trade and Cournot competition

4.5.1 The Solution

When trade is opened the structure of exogenous parameters change in the following aspects: There exist (except the knife edge cases of technological symmetry and wage symmetry or symmetry in overall cost per unit) comparative cost advantages for one good for each country. Factor endowments of labor Λ_h and Λ_f may be different, factor markets are segregated and labor can not move internationally. Product markets for both world markets are integrated, thus the system changes from a duopoly case to a quadropoly with two firms in each country. Firm profits are again split up in a lump sum[26] fashion with demand being represented by aggregated demand like

[26] In detail, profits are distributed in a *national* lump sum fashion. This feature will be discussed in section 4.5.2

in section 4.2.2. Of course, it is later possible to calculate national income and trade pattern due to the knowledge about the production sites of production and the implied national wages paid and profits earned. Wages that are different internationally will be the most likely outcome. Tables 4.4 and 4.5 show the necessary equations for the general equilibrium with cournot competition and the 23 endogenous variables $\Psi, p_1, p_2, x_1, x_{h1}, x_{f1}, x_2, x_{h2}, x_{f2}, l_{h1}, l_{f1}, l_{f2}, l_{h2}, w_h, w_f, \lambda, \Pi_1, \Pi_2,$
$\Pi_h 1, \Pi_{h2}, \Pi_{f1}, \Pi_{f2}$ and Π, where p_1 is again the numéraire. Supply has been calculated in section 4.3.1.2.

Solving this system leads to a much longer solution formulation, but still we only obtain one single global solution, which can be considered quite a success and this is mostly to the well-defined demand properties with linear demand, quadratic preferences and a given and perceived demand for firms. The general solution with no exogenous variable fixed can be review in Appendix 2.3.1. Assuming the same values $a_1 = a_2 = 80, b_1 = b_2 = 10$ as in the autarky case and splitting the labor force equally to $\Lambda_h = \Lambda_f = 50$ gives the following results for the equilibrium production of goods 1

Table 4.4: Trade with Cournot competition

2 product markets	
demand	$x_1 = \frac{a_1 - p_1\lambda}{b_1}$
	$x_2 = \frac{a_2 - p_2\lambda}{b_2}$
supply	$x_{h1} = 2\frac{a_1 + 2w_f\alpha_{f1}\lambda - 3w_h\alpha_{h1}\lambda}{5b_1}$
	$x_{h2} = 2\frac{a_2 + 2w_f\alpha_{f2}\lambda - 3w_h\alpha_{h2}\lambda}{5b_2}$
	$x_{f1} = 2\frac{a_1 + 2w_h\alpha_{h1}\lambda - 3w_f\alpha_{f1}\lambda}{5b_1}$
	$x_{f2} = 2\frac{a_2 + 2w_h\alpha_{h2}\lambda - 3w_f\alpha_{f2}\lambda}{5b_2}$
	$x_{h11} = x_{h12} = \frac{x_{h1}}{2}$
	$x_{h21} = x_{h22} = \frac{x_{h2}}{2}$
	$x_{f11} = x_{f12} = \frac{x_{f1}}{2}$
	$x_{f21} = x_{f22} = \frac{x_{f2}}{2}$
	$x_1 = x_{h1} + x_{f1}$
	$x_2 = x_{h2} + x_{f2}$

factor market	
demand	$l_{h1} = \alpha_{h1}w_h$
	$l_{h2} = \alpha_{h2}w_w$
	$l_{f1} = \alpha_{f1}w_f$
	$l_{f2} = \alpha_{f2}w_f$
supply	Λ_h
	Λ_f
market clearing	$\Lambda_h = l_{h1} + l_{h2}$
	$\Lambda_f = l_{f1} + l_{f2}$

Table 4.5: Trade with Cournot competition continued

	income equations
consumers	$\Phi = w_h \Lambda_h + w_f \Lambda_f + \Pi + \Omega$
	$\Omega = 4F_h + 4F_f$
firms	$\Pi = \Pi_h + \Pi_f$
	$\Pi_{h1} = (p_1 - c_{h1})x_{h1} - 2F_h$
	$\Pi_{h2} = (p_2 - c_{h2})x_{h2} - 2F_h$
	$\Pi_{f1} = (p_1 - c_{f1})x_{f1} - 2F_f$
	$\Pi_{f2} = (p_2 - c_{f2})x_{f2} - 2F_f$
	$\Pi_h = \Pi_{h1} + \Pi_{h2}$
	$\Pi_f = \Pi_{f1} + \Pi_{f2}$
	$c_{h1} = \alpha_{h1} w_h$
	$c_{h2} = \alpha_{h2} w_h$
	$c_{f1} = \alpha_{f1} w_f$
	$c_{f2} = \alpha_{f2} w_f$

and 2 in h and f:[27]

$$x_{h1} = \frac{\alpha f1^2\left(\alpha h1(250-16\alpha h2)+48\alpha h2^2\right)-4\alpha f1\alpha h2(\alpha f2(8\alpha h1-8\alpha h2+50)+75\alpha h2)}{\alpha f1^2\left(5\alpha h1^2+9\alpha h2^2\right)-8\alpha f1\alpha f2\alpha h1\alpha h2+\alpha f2^2\left(9\alpha h1^2+5\alpha h2^2\right)}$$
$$+\frac{2\alpha f2\left(6(25-4\alpha f2)\alpha h1\alpha h2+225\alpha f2\alpha h1+8\alpha f2\alpha h2^2\right)}{\alpha f1^2\left(5\alpha h1^2+9\alpha h2^2\right)-8\alpha f1\alpha f2\alpha h1\alpha h2+\alpha f2^2\left(9\alpha h1^2+5\alpha h2^2\right)} \quad (4.65)$$

$$x_{h2} = \frac{2\alpha f1^2(8\alpha h1(\alpha h1-3\alpha h2)+225\alpha h2)+4\alpha f1\alpha h1(\alpha f2(8\alpha h1-8\alpha h2-50)+75\alpha h2)}{\alpha f1^2\left(5\alpha h1^2+9\alpha h2^2\right)-8\alpha f1\alpha f2\alpha h1\alpha h2+\alpha f2^2\left(9\alpha h1^2+5\alpha h2^2\right)}$$
$$+\frac{2\alpha f2\left(6(4\alpha f2-25)\alpha h1^2+\alpha f2(125-8\alpha h1)\alpha h2\right)}{\alpha f1^2\left(5\alpha h1^2+9\alpha h2^2\right)-8\alpha f1\alpha f2\alpha h1\alpha h2+\alpha f2^2\left(9\alpha h1^2+5\alpha h2^2\right)} \quad (4.66)$$

$$x_{f1} = \frac{50\alpha f1\left(5\alpha h1^2+9\alpha h2^2\right)+4\alpha f2^2\left(8\alpha h1\alpha h2+3\alpha h1(4\alpha h1-25)+4\alpha h2^2\right)}{\alpha f1^2\left(5\alpha h1^2+9\alpha h2^2\right)-8\alpha f1\alpha f2\alpha h1\alpha h2+\alpha f2^2\left(9\alpha h1^2+5\alpha h2^2\right)}$$
$$-\frac{4\alpha f2\left(\alpha f1\left(4\alpha h1^2+8\alpha h1\alpha h2+3\alpha h2(4\alpha h2-25)\right)+50\alpha h1\alpha h2\right)}{\alpha f1^2\left(5\alpha h1^2+9\alpha h2^2\right)-8\alpha f1\alpha f2\alpha h1\alpha h2+\alpha f2^2\left(9\alpha h1^2+5\alpha h2^2\right)} \quad (4.67)$$

$$x_{f2} = \frac{4\alpha f1^2\left(4\alpha h1^2+8\alpha h1\alpha h2+3\alpha h2(4\alpha h2-25)\right)+50\alpha f2\left(9\alpha h1^2+5\alpha h2^2\right)}{\alpha f1^2\left(5\alpha h1^2+9\alpha h2^2\right)-8\alpha f1\alpha f2\alpha h1\alpha h2+\alpha f2^2\left(9\alpha h1^2+5\alpha h2^2\right)}$$
$$-\frac{4\alpha f1\left(\alpha f2\left(8\alpha h1\alpha h2+3\alpha h1(4\alpha h1-25)+4\alpha h2^2\right)+50\alpha h1\alpha h2\right)}{\alpha f1^2\left(5\alpha h1^2+9\alpha h2^2\right)-8\alpha f1\alpha f2\alpha h1\alpha h2+\alpha f2^2\left(9\alpha h1^2+5\alpha h2^2\right)} \quad (4.68)$$

A few plots of this scenario help to understand the equations (4.65) to (4.68) in detail and check, whether they are intuitively correct. Figure 4.2 shows a 3D plot of home production with volume on the vertical z axis and technology parameters from 0 to 50 on the x and y axis.

Given the case that the production of x_{h1} is relatively cheap ($\alpha_{h1} < \alpha_{h2}$ at the same w_h), the red plane is above the green one (which would be on the right side of the plot). Comparing the produced amount of one good (x_1) between countries, so x_{h1} and x_{f1} gives the 3D plot presented in figure 4.3.

Obviously, also between countries, technology parameters have to be rather similar to allow a feasible general equilibrium. Figure 4.3 makes perfectly economic sense again: If production of good 1 at home is relatively cheap ($\alpha_{h1}/\alpha_{h2} < \alpha_{f1}/\alpha_{f2}$), i.e.

[27]Mind again the slightly different notation in lower indexes from Mathematica.

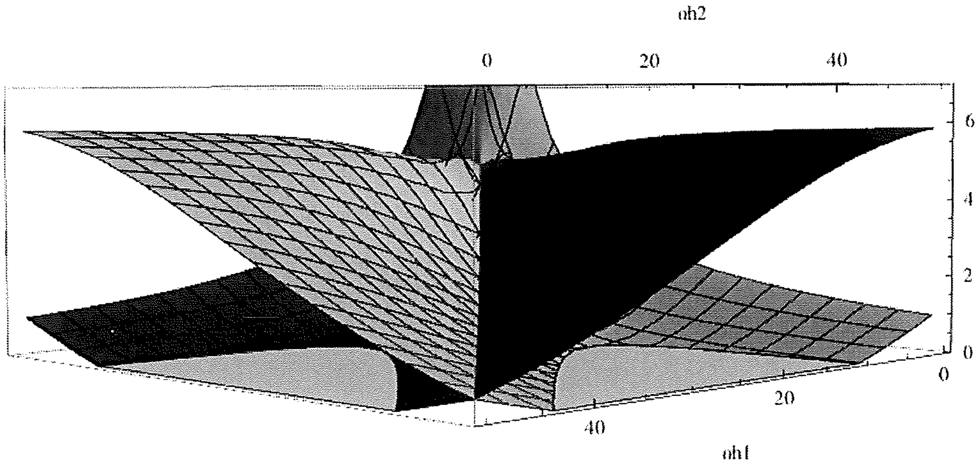

Figure 4.2: Volume of production in h with x_{h1} (red) and x_{h2} (green) at $\alpha_{f1} = \alpha_{f2} = 30$.

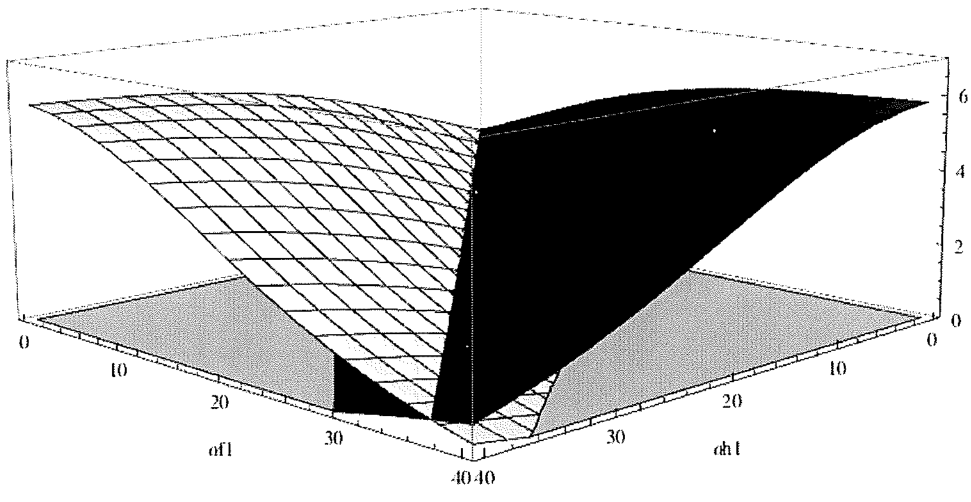

Figure 4.3: Volume of production of x_1 with x_{h1} (red) and x_{f1} (blue) at $\alpha_{h2} = 33$ and $\alpha_{f2} = 30$.

there exists a comparative cost advantage for good one at home, home production will be larger than foreign production: The red plane is above the blue one and vice versa. But why don't we get a complete specialization in the absence of a curved production possibility frontier? *Imperfect competition* allows foreign firms to stay in sector one even though they have a comparative cost disadvantage and face constant returns to scale in production. Their profits may shrink but they are still high enough to cover marginal and fixed cost and make non-negative profits. Also mind the detail in figure 4.3 that the front-rear axis of equal production at home and abroad is slightly shifted to the left. This is due to the fact that $\alpha_{h2} = 33$ and $\alpha_{f2} = 30$, so the home country has an absolute disadvantage in good two, which has to be compensated in the α_{h1} and α_{f1} system. To calculate the threshold of positive production of good one abroad (x_{f1}), we start at:

$$\begin{aligned} x_{f1} &> 0, \quad \text{or} \\ 0 &< \frac{50\alpha\mathrm{f1}\left(5\alpha\mathrm{h1}^2+9\alpha\mathrm{h2}^2\right)+4\alpha\mathrm{f2}^2\left(8\alpha\mathrm{h1}\alpha\mathrm{h2}+3\alpha\mathrm{h1}(4\alpha\mathrm{h1}-25)+4\alpha\mathrm{h2}^2\right)}{\alpha\mathrm{f1}^2\left(5\alpha\mathrm{h1}^2+9\alpha\mathrm{h2}^2\right)-8\alpha\mathrm{f1}\alpha\mathrm{f2}\alpha\mathrm{h1}\alpha\mathrm{h2}+\alpha\mathrm{f2}^2\left(9\alpha\mathrm{h1}^2+5\alpha\mathrm{h2}^2\right)} \\ &\quad -\frac{4\alpha\mathrm{f2}\left(\alpha\mathrm{f1}\left(4\alpha\mathrm{h1}^2+8\alpha\mathrm{h1}\alpha\mathrm{h2}+3\alpha\mathrm{h2}(4\alpha\mathrm{h2}-25)\right)+50\alpha\mathrm{h1}\alpha\mathrm{h2}\right)}{\alpha\mathrm{f1}^2\left(5\alpha\mathrm{h1}^2+9\alpha\mathrm{h2}^2\right)-8\alpha\mathrm{f1}\alpha\mathrm{f2}\alpha\mathrm{h1}\alpha\mathrm{h2}+\alpha\mathrm{f2}^2\left(9\alpha\mathrm{h1}^2+5\alpha\mathrm{h2}^2\right)}, \end{aligned}$$

Reducing this term in Mathematica with respect to α_{h1}, α_{f1} :

$$\alpha\mathrm{h1} > 0 \wedge 0 < \alpha\mathrm{f1} < \frac{720(2178+\alpha\mathrm{h1}(67+6\alpha\mathrm{h1})}{78111+\alpha\mathrm{h1}(3168+23\alpha\mathrm{h1})} \tag{4.69}$$

In this non-linear setting it is necessary to calculate all four thresholds for the exogenous technology parameters with given other exogenous variables to obtain the set of feasible general equilibrium locations.

A very important fact is that - as it has already been mentioned before - wherever the blue plane on the bottom is visible, one of the production goods would have a negative equilibrium volume in home production. Again, these are not feasible settings, there is no economic sense.[28] Therefore, one can argue that one finding would be:

Hypothesis 2. *Only if technology parameters (α_{ij}) are rather similar countrywise, trade will lead to a feasible general equilibrium solution.*

[28]The same results occur for all different numerical examples tested.

An other figure to illustrate hypothesis 2 is the plot of figure 4.4:

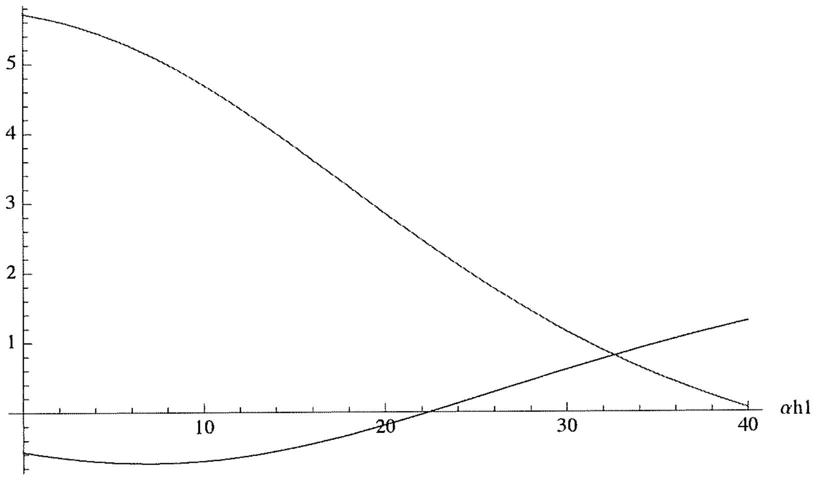

Figure 4.4: Volume of production of x_1 with x_{h1} (red) and x_{f1} (blue) at $\alpha_{f1} = 30$, $\alpha_{h2} = 33$ and $\alpha_{f2} = 30$.

This is a cross section of figure 4.3 at $\alpha_{f1} = 30$. Obviously, the home technology parameter for production of good one (α_{h1}) has to be in between 22 and 40 to guarantee a feasible general equilibrium. At $\alpha_{h1} \approx 33$, the lines cross because here no country has a comparative cost advantage, which means that both countries produce in the same amount of good one. The exact values can be calculated with the equilibrium wages from h and f. That is, solving the general equilbrium with $\alpha_{h2} = 33, \alpha_{f1} = \alpha_{f2} = 30$ and the demand and endowment factors as before, the equilibrium quantitites of x_{h1} and x_{f1} are:

$$\begin{aligned} x_{h1} = \frac{43527 - 1069\alpha\text{h}1}{\alpha\text{h}1(7\alpha\text{h}1 - 132) + 7623} &= \frac{\alpha\text{h}1(121\alpha\text{h}1 - 1560) - 25839}{6(\alpha\text{h}1(7\alpha\text{h}1 - 132) + 7623)} = x_{f1}, \\ \alpha_{h1} &= 32.613075155. \end{aligned} \tag{4.70}$$

This implies a wage at home of $w_h = 0.0287314$ and abroad of $w_f = 0.031234$ that offsets the absolute cost advantage in good one for country f.

The following 3D plot in figure 4.5 shows the production of all four firm types in the two sectors. Obviously, both good markets show a higher production in h if the corresponding α_{hj} are lower, i.e. the home country is more productive. This plot shows again the claim of hypothesis 2.

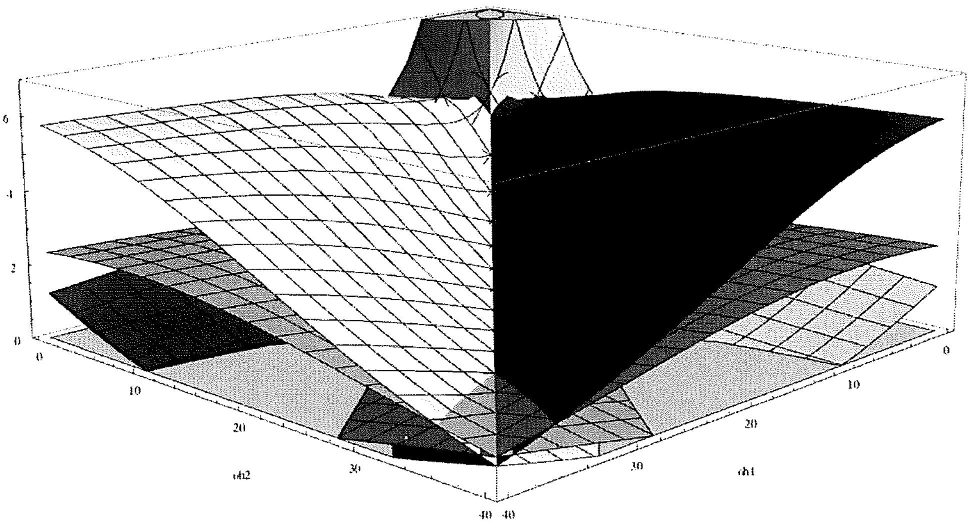

Figure 4.5: Volume of production of x_{h1} (red), x_{h2} (light blue), x_{f1} (green) and x_{f2} (brown) at $\alpha_{f1} = 30$ and $\alpha_{f2} = 30$.

Figure 4.6 shows that the production of a certain good correlates positively with the production of the other good in the other country, which indicates beneficial specialization and trade of the relatively advantageous good. The black line is total production, showing that Ricardo is still right: with a comparative cost advantage in good one, total (world) output is highest if the home technology parameter for good one (α_{h1}) is on the border of feasability and total specialization occurs.

4.5.2 The Effects of Trade

There are different effects that affect the general equilibrium when moving from autarky to trade: *On the one hand*, the two product market duopolies become quadropolies. This, as shown in f.e. (Ruffin 2003*a*), should increase output and welfare due to a competition effect. *On the other hand*, complete specialization, which would be most productive in terms of linear production cost, would lead again to national duopolies serving the integrated world market for the two goods. Therefore, the hypothesis is that

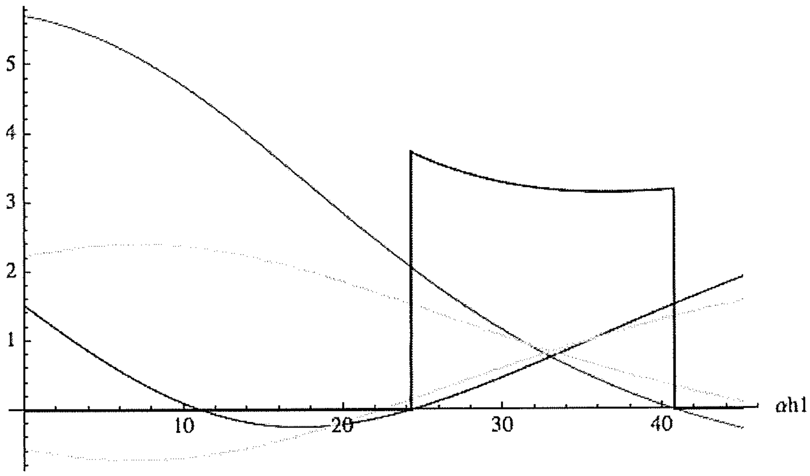

Figure 4.6: Volume of production of x_{h1} (red), x_{h2} (blue), x_{f1} (green), x_{f2} (orange) and positive total production $x_1 + x_2 \wedge x_{lj} > 0$ (black/filled) at $\alpha_{f1} = 30$, $\alpha_{h2} = 33$ and $\alpha_{f2} = 30$.

the highest output will be produced when the firms of the country with the competitive disadvantage just stay in market, i.e. their profits are zero and thus both sectors remain in a quadropoly. I call this the *competition-focused specialization.* This is a necessary addition to the comparative cost of (Ricardo 1817) and its further investigation in modern economics by f.e. (Samuelson 1964) and in an oligopolistic equilibrium in (Ruffin 2003*b*) and (Neary 2007).

The rest of this section will compare the following three cases: Autarky, complete and competition-friendly specialization. The approach is to take the used parametrization for endowments and exogenous factors and let some technology parameters move freely to determine the thresholds for non-negative profits and production. First, three of the four technology parameters are being set. In this case to $\alpha_{h1} = \alpha_{f1} = \alpha_{f2}$. All three technology parameters are fixed to the same value to see the comparative advantage in a changing α_{h2} more easily. The next step is to search for a general equilibrium in the Cournot and autarky case and fix all exogenous parameters except the important α_{h2}. Later, this will allow to search for threshold levels of this α_{h2} that make home of foreign profits in sectors one or two negative, which would drop out home or foreign firms in that industry respectively. The last step is to calculate and compare income Ψ, wages w_h & w_f and profits Π_h & Π_f of the competition friendly scenario at these thresholds to obtain information whether it is more beneficial than the case of complete specialization and/or autarky.

The setting of exogenous parameters is the following:

$$\begin{pmatrix} b_1 = 10 & b_2 = 10 & a_1 = 80 \\ a_2 = 80 & \Lambda_h = 50 & \Lambda_f = 50 \\ \Lambda = 100 & F_h = 0.001 & F_f = 0.001 \\ \alpha_{h1} = 30 & \alpha_{f1} = 30 & \alpha_{f2} = 30 \end{pmatrix}$$

Given these parameters the autarky and cournot general equilibria from tables 4.2 and 4.4 - 4.5 respectively can be solved. For the cournot quadropoly case, the four profit functions, depending on α_{h2} only, are:

$$\Pi_{h1} = \frac{21493}{3500} + \frac{206850 - 20\alpha h2}{44100 - 840\alpha h2 + 49\alpha h2^2} - \frac{50(11025 + 344\alpha h2)}{52500 + \alpha h2(760 + 43\alpha h2)}, \tag{4.71}$$

$$\Pi_{h2} = -\frac{1}{500} - \frac{70(285 + 8\alpha h2)}{6300 + \alpha h2(-120 + 7\alpha h2)} + \frac{368550 + 3440\alpha h2}{52500 + \alpha h2(760 + 43\alpha h2)}, \tag{4.72}$$

$$\Pi_{f1} = \frac{8956}{37625} + \frac{33600 - 1970\alpha h2}{44100 - 840\alpha h2 + 49\alpha h2^2} + \frac{8610(-115 + 8\alpha h2)}{43(52500 + \alpha h2(760 + 43\alpha h2))}, \tag{4.73}$$

$$\Pi_{f2} = \frac{456854}{338625} + \frac{33600 - 1970\alpha h2}{44100 - 840\alpha h2 + 49\alpha h2^2} + \frac{170(-248955 + 586\alpha h2)}{387(52500 + \alpha h2(760 + 43\alpha h2))}. \tag{4.74}$$

The corresponding plots are shown in figure 4.7. Intuitively, the higher α_{h2} the higher are profits for home firms in sector one (Π_{h1} - red) and foreign profits in sector 2 (Π_{f2} - brown) and vice versa for Π_{h2} (green) and Π_{f1} (blue). Obviously, profits are highest at the boundaries of the (feasible) middle part that is shaded.

The question is now where do these profits line cross the x-axis, i.e. where are the non-negativity threshold levels for firm profits in the given system. To calculate this, we take (4.71) to (4.74), set them ≥ 0 and then solve for α_{h2}. This gives exactly that (exogenous) levels of α_{h2}, where firms start to make non-negative profits, which means

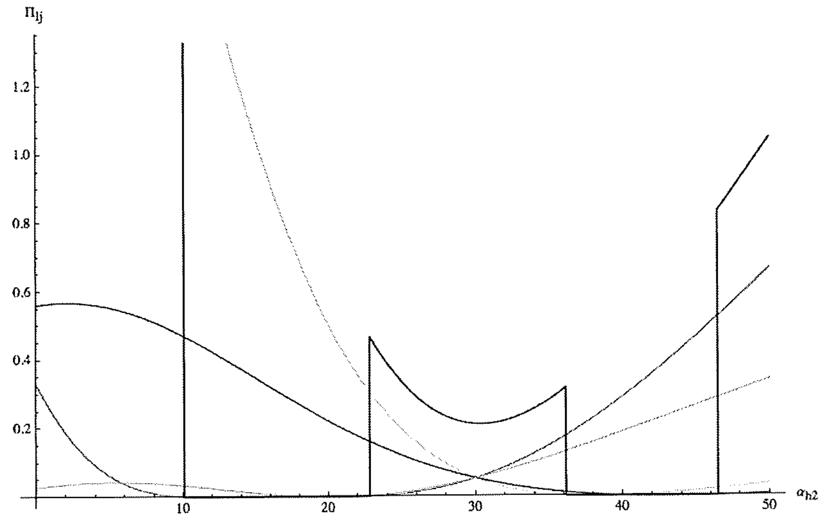

Figure 4.7: Profits of Π_{h1} (red), Π_{h2} (green), Π_{f1} (blue), Π_{f2} (brown) and total profits Π (black) at $\alpha_{h1} = 30$, $\alpha_{f1} = 30$ and $\alpha_{f2} = 30$.

survive in the given general oligopolistic equilibrium. This is:

$$\Pi_{h1} \geq 0 \quad \rightarrow \quad \alpha_{h2} \geq 22.7837, \tag{4.75}$$

$$\Pi_{h2} \geq 0 \quad \rightarrow \quad \alpha_{h2} \leq 36.1295, \tag{4.76}$$

$$\Pi_{f1} \geq 0 \quad \rightarrow \quad \alpha_{h2} \leq 40.1257, \tag{4.77}$$

$$\Pi_{f2} \geq 0 \quad \rightarrow \quad \alpha_{h2} \geq 22.0262. \tag{4.78}$$

From the two lower and two upper bounds for α_{h2} in (4.75) to (4.78), the smaller (and therefore relevant) boundaries determine that:

$$22.7837 \leq \alpha_{h2} \leq 36.1295. \tag{4.79}$$

Only if (4.79) is fulfilled both home and foreign firms in both sectors operate and we have two Cournot quadropolies.

The next step is to ask the following question: *Starting at maximum competition friendly specialization, that is* $\alpha_{h2} = 22.7837$ *or* $\alpha_{h2} = 36.1295$, *does a shift towards a more efficient production by producing the goods only in the country with the com-*

parative advantage lead to an increase or decrease of income?[29] To do so, we need another form of international equilibrium, one with total of production of one good in the comparatively advantageous country, or $x_{h1} = x_{f2} = 0$ if $\alpha_{h1}/\alpha_{h2} > \alpha_{f1}/\alpha_{f2}$ or $x_{h2} = x_{f1} = 0$ if $\alpha_{h1}/\alpha_{h2} < \alpha_{f1}/\alpha_{f2}$. This only changes the supply side of products in tables 4.4 - 4.5 to:

$$
\begin{aligned}
x_1 = 2\frac{a_1 - w_h\alpha_{h1}\lambda}{3b_1} &\wedge x_2 = 2\frac{a_2 - w_f\alpha_{f2}\lambda}{3b_2}, \qquad (4.80)\\
\text{if } \alpha_{h1}/\alpha_{h2} &> \alpha_{f1}/\alpha_{f2},\\
x_1 = 2\frac{a_1 - w_f\alpha_{f1}\lambda}{3b_1} &\wedge x_2 = 2\frac{a_2 - w_h\alpha_{h2}\lambda}{3b_2}, \qquad (4.81)\\
\text{if } \alpha_{h1}/\alpha_{h2} &< \alpha_{f1}/\alpha_{f2}.
\end{aligned}
$$

Additionally, sunk cost shrink by $2F_h + 2F_f$ since the number of firms drops from eight to four. The mathematica setting of this can be found in appendix 7.

The solution for $\alpha_{h1} = \alpha_{f1} = \alpha_{f2} = 30, \alpha_{h2} = 22.7837$ is:

Autarky

$$
\begin{pmatrix}
\Psi \to 3.57592 & p_2 \to 0.858084 & x_1 \to 1.49611 & x_2 \to 2.41912 \\
l_1 \to 44.8833, & l_2 \to 55.1167 & w \to 0.0294994 & \lambda \to 65.0389 \\
\Pi_1 \to 0.170078 & \Pi_2 \to 0.447894 & \Pi \to 0.617972 & p_1 \to 1
\end{pmatrix}
$$

[29]The term income in this matter should be taken as an equivalent to measure wealth and welfare. The distribution of income between wages, profits and fixed cost/investment at home and abroad are subject to the analysis of the Stackelberg equilibrium.

Competition focused specialization

$$\begin{pmatrix} \Psi \to 3.60063 & p_2 \to 0.904987 & x_1 \to 1.60129 & x_{h1} \to 0.159992 \\ x_{f1} \to 1.4413 & x_2 \to 2.20925 & x_{h2} \to 1.98388 & x_{f2} \to 0.225372 \\ l_{h1} \to 4.79975 & l_{f1} \to 43.2389 & l_{f2} \to 6.76115 & l_{h2} \to 45.2003 \\ w_h \to 0.0329166 & w_f \to 0.0295792 & \lambda \to 63.9871 & \Pi_1 \to 0.160324 \\ \Pi_2 \to 0.307513 & \Pi_{h1} \to 0 & \Pi_{h2} \to 0.305544 & \Pi_{f1} \to 0.160324 \\ \Pi_{f2} \to 0.00196895 & \Pi \to 0.467837 & p_1 \to 1 & \end{pmatrix}$$

Full specialzation

$$\begin{pmatrix} \Psi \to 3.67796 & p_2 \to 0.916746 & x_1 \to 1.66667 & x_2 \to 2.19394 \\ l_{f1} \to 50 & l_{h2} \to 50 & w_h \to 0.0326257 & w_f \to 0.0289474 \\ \lambda \to 63.3333 & \Pi_{f1} \to 0.217298 & \Pi_{h2} \to 0.378005 & \Pi \to 0.595303 \\ p_1 \to 1 & & & \end{pmatrix}$$

In the second boundary case with $\alpha_{h2} = 36.1295$, the income values are $\Psi_{autarky} = 3.2308$, $\Psi_{competitionfocused} = 3.40415$ and full specialization (all x_1 in h and all x_2 in f):

$$\begin{pmatrix} \Psi \to 3.33333 & p_2 = p_1 \to 1 & x_1 \to 1.66667 & x_2 \to 1.66667 \\ l_{h1} \to 50 & l_{f2} \to 50 & w_h \to 0.0289474 & w_f \to 0.0289474 \\ \lambda \to 63.3333 & \Pi_{h1} \to 0.217298 & \Pi_{f2} \to 0.217298 & \Pi \to 0.434596 \end{pmatrix}$$

This leads to the result that the old Ricardian idea of comparative cost advantages and specialization still remain under oligopoly, as also noted by (Samuelson 1964) and (Ruffin 2003*b*). The latter case of $\alpha_{h2} = 36.1295$ is of course one, where no absolute cost advantages can be relatively exploited (since the weaker good is not produced at all, whereas it is exploited in the first case when $\alpha_{h2} = 22.7837$ and that is > 30). This case even shows both cases, the first one shows the importance of the comparative cost advantage of trade by clearly stating: The more trade and specialization (even with loss of competition), the higher the income. The second case draws a different picture: The loss of competition cannot be compensated by a relatively more efficient production, when countries fully specialize since in the second case $\Psi_{competitionfocused} = 3.40415 > \Psi_{fullyspecialized} = 3.33333$, but this is due to the fact that there exist no absolute cost advantages in good x_1, so this result is only determined by a competition effect without the *ability* to produce more effectively overseas. Therefore:

Hypothesis 3. *If there exist comparative cost advantages, full specialization has a higher (positive) impact on income Ψ than the (negative) loss of competition. Absolute cost advantages in at least one good (non-identical technology for the two countries) are a necessary condition to utilize this positive effect of specialization. In any case, trade leads to an increase in income, either by fostering competition (even in the absence of absolute advantages) or by exploiting productivity gains (in the absence of increased competition).*

Findings for hypothesis 3 second (Samuelson 1964), since in most cases absolute cost advantages in at least one good exist, i.e. countries are different in their technology.

4.6 Equilibrium and Market Leadership

4.6.1 One Leader and Three Followers

If one of the home firms choses to invest G_f to speed up the process of gathering information on foreign fixed cost, the system enters Stackelberg competition as explained in detail in section 4.1.2. This is in this form a novelty to the general oligopolistic equilibrium, especially in a case that can be closed due to the properties of feedback limitation, linear and perceived demand. Before going to the actual calculations, one may ask what the expected outcome could be: *First*, since demand is linear the Stackelberg case should lead to a higher equilibrium quantity and lower equilibrium price just like in the partial equilibrium case. *Second*, the higher equilibrium product quantity asks for more labor, the input factor, which does of course not change in endowment. *Third*, profits of the new leader (may) rise but profits of the followers should shrink, therefore we cannot see a clear effect on profits and therefore also not on the lump sum income which consists of profits Π and fixed cost/investment Ω_S, which rises by G_f by the multinational industry leader.[30]

Leaving the detailed analysis of the Cournot vs. the one or two leader Stackelberg case up to section 4.6.3, the one leader case uses demand in the same way as before, the supply side of countries h and f are now differently organized and firms optimize according to the setting in section 4.3.1.3. That sets up the general oligopolistic equilibrium in tables 4.6 and 4.7. The general solution, which is getting much longer due to the Stackelberg competition, can be reviewed in appendix 2.3.1.

Again, a few plots help us to understand the result. Figure 4.8 shows the same picture as Cournot competition: Ricardian cost advantages and subsequent specialization yield highest outputs (left boundary of the feasible set/shaded area).

Doing the same calculation on thresholds of positive production for all firm types as in section 4.5.2 shows that in the Stackelberg case with given exogenous parameters except α_{h2} has a smaller interval on home technology in sector 2. This is due to the weaker power of sector one at home, which now faces high demand for labor by the Stackelberg industry 2 and has a relative disadvantage with a low α_{h2}. Also wages have to be calculated to search for changes from Cournot competition.

$$\alpha_{h2}^{Stackelberg1leader} \in \left]25.0981; 37.9787\right[\;\Leftrightarrow\; x_{lj} > 0 \text{ for } j = 1, 2; l = h, f \tag{4.82}$$

$$\alpha_{h2}^{Cournot} \in \left]22.7837; 36.1295\right[\;\Leftrightarrow\; x_{lj} > 0 \text{ for } j = 1, 2; l = h, f \tag{4.83}$$

The outcome also confirms intuition on equilibrium quantities. Figure 4.9 shows that the leader always has a larger output than the follower, which must not be mistaken for non-existence of a second-mover advantage, which depends on profits and not quantities. A sufficiently high G_f may show a second mover advantage in section 4.6.4. Additionally to (dis)advantages due to given endowments of labor and/or technology parameters though, the follower in h may face a higher factor price due to high

[30] I will continue to use the term *industry leader*, although Stackelberg himself has always spoken of the *independent* position, whereas the followers were describes as the *dependent* firms, so also the German original (von Stackelberg 1934) never spoke about a *Marktführer* and *-folger*.

Table 4.6: Trade with one Stackelberg leader

2 product markets	
demand	$x_1 = \frac{a_1 - p_1\lambda}{b_1}$
	$x_2 = \frac{a_2 - p_2\lambda}{b_2}$
supply	$x_{h1} = 2\frac{a_1 + 2w_f\alpha_{f1}\lambda - 3w_h\alpha_{h1}\lambda}{5b_1}$
	$x_{h21}^{leader} = \frac{a_2 + 2w_f\alpha_{f2}\lambda - 3w_h\alpha_{h2}\lambda}{2b_2}$
	$x_{h22}^{follower} = \frac{a_2 + 2w_f\alpha_{f2}\lambda - 3w_h\alpha_{h2}\lambda}{8b_2}$
	$x_{f1} = 2\frac{a_1 + 2w_h\alpha_{h1}\lambda - 3w_f\alpha_{f1}\lambda}{5b_1}$
	$x_{f2}^{follower} = 2\frac{a_2 - 6w_f\alpha_{f2}\lambda + 5w_h\alpha_{h2}\lambda}{8b_2}$
	$x_{h11} = x_{h12} = \frac{x_{h1}}{2}$
	$x_{h2} = x_{h21}^{leader} + x_{h22}^{follower}$
	$x_{f11} = x_{f12} = \frac{x_{f1}}{2}$
	$x_{f21} = x_{f22} = \frac{x_{f2}^{follower}}{2}$
	$x_1 = x_{h1} + x_{f1}$
	$x_2 = x_{h2} + x_{f2}$

factor market	
demand	$l_{h1} = \alpha_{h1}w_h$
	$l_{h2} = \alpha_{h2}w_w$
	$l_{f1} = \alpha_{f1}w_f$
	$l_{f2} = \alpha_{f2}w_f$
supply	Λ_h
	Λ_f
market clearing	$\Lambda_h = l_{h1} + l_{h2}$
	$\Lambda_f = l_{f1} + l_{f2}$

Table 4.7: Trade with one Stackelberg leader continued

	income equations
consumers	$\Phi = w_h\Lambda_h + w_f\Lambda_f + \Pi + \Omega$
	$\Omega = 4F_h + 4F_f + G_f$
firms	$\Pi = \Pi_h + \Pi_f$
	$\Pi_{h1} = (p_1 - c_{h1})x_{h1} - 2F_h$
	$\Pi_{h21}^{leader} = (p_2 - c_{h2})x_{h21} - F_h - G_f$
	$\Pi_{h22}^{follower} = (p_2 - c_{h2})x_{h22} - F_h$
	$\Pi_{f1} = (p_1 - c_{f1})x_{f1} - 2F_f$
	$\Pi_{f2}^{follower} = (p_2 - c_{f2})x_{f2} - 2F_f$
	$\Pi_h = \Pi_{h1} + \Pi_{h2}$
	$\Pi_{h2} = \Pi_{h21}^{leader} + \Pi_{h22}^{follower}$
	$\Pi_f = \Pi_{f1} + \Pi_{f2}$
	$c_{h1} = \alpha_{h1}w_h$
	$c_{h2} = \alpha_{h2}w_h$
	$c_{f1} = \alpha_{f1}w_f$
	$c_{f2} = \alpha_{f2}w_f$

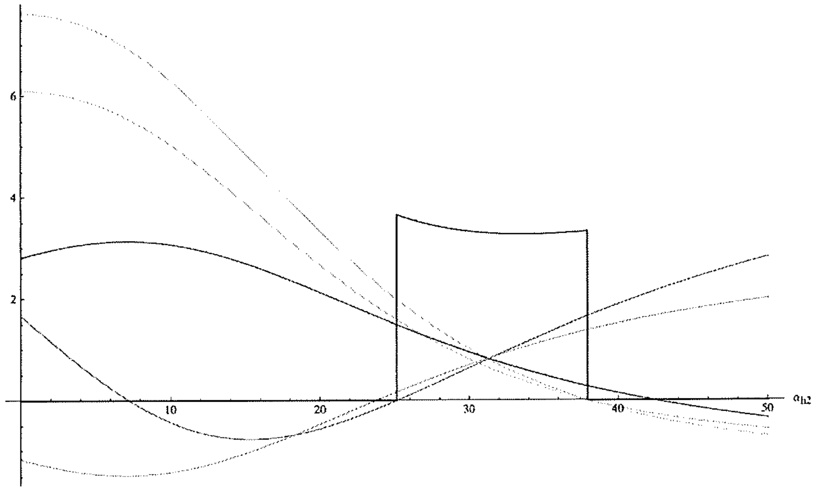

Figure 4.8: Equilibrium quantities of x_{h1} (red), x_{h21}^{leader} (green), $x_{h22}^{follower}$ (yellow), x_{h2} (orange), x_{f1} (blue), $x_{f2}^{followers}$ (brown) and total production $x_1 + x_2 \wedge x_{lj} > 0$ (black/shaded) at $\alpha_{h1} = \alpha_{f1} = \alpha_{f2} = 30$.

demand for labor by the industry leader. This will also be discussed in detail section 4.6.3 on the effects on market leadership.

4.6.2 Two Leaders and Two Followers

If both firms in sector 2 in h decide to undergo foreign direct investment and thus gain informational advantage on fixed cost in the other country, the system is one with Stackelberg competition between two leaders and two followers, where both leaders and both followers decide on their quantities simultaneously respectively.

Leaving the strategy and general equilibrium analysis to the following sections, the results in the manner of the previous sections are: Appendix 2.3.1 shows the general solution. For given and unchanged exogenous parameters from previous sections, the corresponding plots show a typical result. Figure 4.10 shows that the leading country h can produce more of good two, even in general equilibrium, even if there exists a comparative advantage by country f.

A very interesting plot and calculation again is the region of a feasible production set. With the method of previous sections, the possible interval of α_{h2} at $\alpha_{h1} = \alpha_{f1} =$

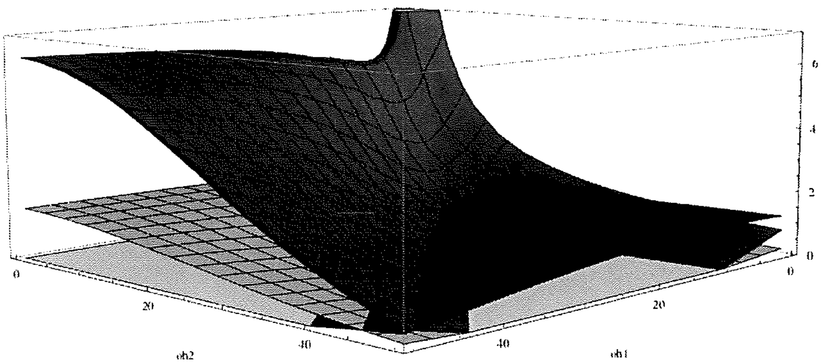

Figure 4.9: Equilibrium quantities of x_{h21}^{leader} (red), $x_{h22}^{follower}$ (green) and $x_{f2}^{followers}$ (blue) at $\alpha_{f1} = \alpha_{f2} = 30$.

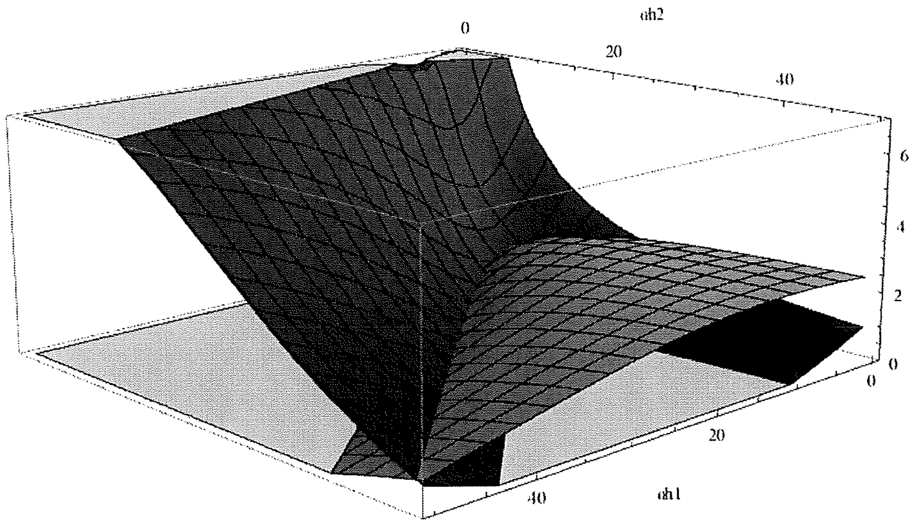

Figure 4.10: Equilibrium quantities of $x_{h2}^{leaders}$ (red) and $x_{f2}^{followers}$ (green) at $\alpha_{f1} = \alpha_{f2} = 30$.

Table 4.8: Trade with two Stackelberg leaders

2 product markets

demand	$x_1 = \frac{a_1 - p_1\lambda}{b_1}$
	$x_2 = \frac{a_2 - p_2\lambda}{b_2}$
supply	$x_{h1} = 2\frac{a_1 + 2w_f\alpha_{f1}\lambda - 3w_h\alpha_{h1}\lambda}{5b_1}$
	$x_{h2}^{leaders} = 2\frac{a_2 + 2w_f\alpha_{f2}\lambda - 3w_h\alpha_{h2}\lambda}{3b_2}$
	$x_{f1} = 2\frac{a_1 + 2w_h\alpha_{h1}\lambda - 3w_f\alpha_{f1}\lambda}{5b_1}$
	$x_{f2}^{followers} = 2\frac{a_2 - 7w_f\alpha_{f2}\lambda + 6w_h\alpha_{h2}\lambda}{9b_2}$
	$x_{h11} = x_{h12} = \frac{x_{h1}}{2}$
	$x_{h21} = x_{h22} = \frac{x_{h2}^{leaders}}{2}$
	$x_{f11} = x_{f12} = \frac{x_{f1}}{2}$
	$x_{f21} = x_{f22} = \frac{x_{f2}^{follower}}{2}$
	$x_1 = x_{h1} + x_{f1}$
	$x_2 = x_{h2} + x_{f2}$

factor market

demand	$l_{h1} = \alpha_{h1}w_h$
	$l_{h2} = \alpha_{h2}w_w$
	$l_{f1} = \alpha_{f1}w_f$
	$l_{f2} = \alpha_{f2}w_f$
supply	Λ_h
	Λ_f
market clearing	$\Lambda_h = l_{h1} + l_{h2}$
	$\Lambda_f = l_{f1} + l_{f2}$

Table 4.9: Trade with one Stackelberg leader continued

	income equations
consumers	$\Phi = w_h\Lambda_h + w_f\Lambda_f + \Pi + \Omega$
	$\Omega = 4F_h + 4F_f + 2G_f$
firms	$\Pi = \Pi_h + \Pi_f$
	$\Pi_{h1} = (p_1 - c_{h1})x_{h1} - 2F_h$
	$\Pi_{h2}^{leaders} = (p_2 - c_{h2})x_{h2} - 2F_h - 2G_f$
	$\Pi_{f1} = (p_1 - c_{f1})x_{f1} - 2F_f$
	$\Pi_{f2}^{followers} = (p_2 - c_{f2})x_{f2} - 2F_f$
	$\Pi_h = \Pi_{h1} + \Pi_{h2}$
	$\Pi_f = \Pi_{f1} + \Pi_{f2}$
	$c_{h1} = \alpha_{h1}w_h$
	$c_{h2} = \alpha_{h2}w_h$
	$c_{f1} = \alpha_{f1}w_f$
	$c_{f2} = \alpha_{f2}w_f$

$\alpha_{f2} = 30$ is:

$$\alpha_{h2}^{Stackelberg2leader} \in \left]25.4804; 37.9787\right[\;\Leftrightarrow\; x_{lj} > 0 \text{ for } j = 1,2; l = h,f \quad (4.84)$$

compared to previous results of

$$\alpha_{h2}^{Stackelberg1leader} \in \left]25.0981; 37.9787\right[\;\Leftrightarrow\; x_{lj} > 0 \text{ for } j = 1,2; l = h,f,$$

$$\alpha_{h2}^{Cournot} \in \left]22.7837; 36.1295\right[\;\Leftrightarrow\; x_{lj} > 0 \text{ for } j = 1,2; l = h,f.$$

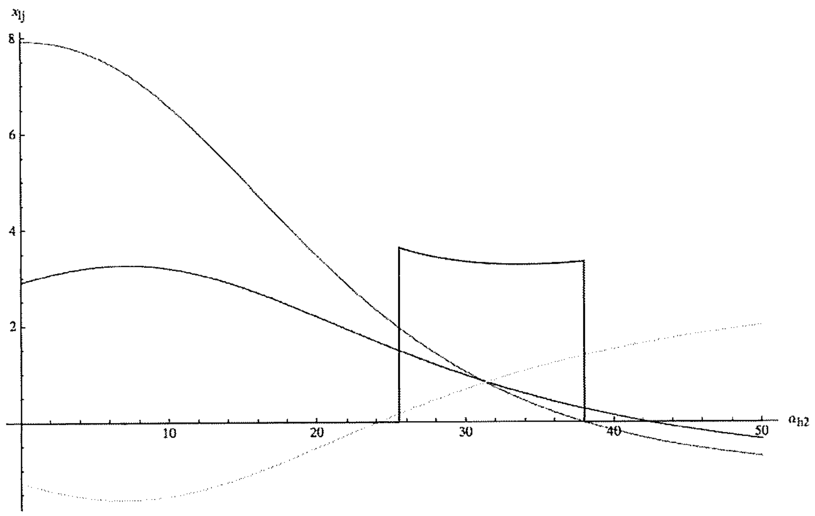

Figure 4.11: Equilibrium quantities of $x_{h2}^{leaders}$ (red), $x_{f2}^{followers}$ (green), x_{h1} (yellow), x_{f1} (blue) and $x_1 + x_2 \wedge x_{lj} > 0$ (black) at $\alpha_{f1} = \alpha_{f2} = 30$.

4.6.3 Effects of Market Leadership

4.6.3.1 Effects on income

I will introduce a slightly different setting of exogenous parameters to highlight the effects of market leadership in this section. They are:

$$\begin{pmatrix} b_1 = 10 & b_2 = 10 & a_1 = 80 & a_2 = 80 & \Lambda_h = 50 \\ \Lambda_f = 50 & F_h = 0.001 & F_f = 0.001 & G_f = 0.0005 & \alpha_{h1} = 30 \\ \alpha_{h2} = 29 & \alpha_{f1} = 32 & \alpha_{f2} = 33 & & \end{pmatrix}$$

Obviously, the home country has a comparative cost advantage (and for sure absolute cost advantage in pure technology terms) in good two. The choice of technology parameters to be rather similar is necessary to obtain feasible results, a variation of preference and demand values and labor endowment does not really the characteristics of the results. The foreign branch, that can be build through foreign direct investment G_f, is cheaper than the headquarter plant at home. This is a common approach like in (Markusen 2004). Section 4.6.4 will discuss threshold levels of fixed cost that make FDI pay off or not. Let us take a look at the general equilibria with the given parameters:

Cournot competition in both sectors:

$$\begin{pmatrix} \Psi \to 3.24409 & p_2 \to 0.998908 & x_1 \to 1.61945 & x_{h1} \to 0.655973 \\ x_{f1} \to 0.963477 & x_2 \to 1.62642 & x_{h2} \to 1.04555 & x_{f2} \to 0.58087 \\ l_{h1} \to 19.6792 & l_{f1} \to 30.8313 & l_{f2} \to 19.1687 & l_{h2} \to 30.3208 \\ w_h \to 0.0316199 & w_f \to 0.0288906 & \lambda \to 63.8055 & \Pi_1 \to 0.102463 \\ \Pi_2 \to 0.108104 & \Pi_{h1} \to 0.0317197 & \Pi_{h2} \to 0.0836639 & \Pi_{f1} \to 0.0707436 \\ \Pi_{f2} \to 0.0244405 & \Pi \to 0.210568 & p_1 \to 1 & \end{pmatrix}$$

Cournot competition in sector one and Stackelberg competition with one leader from h in sector two:

$$\begin{pmatrix} \Psi \to 3.21523 & p_2 \to 0.974633 & x_1 \to 1.54748 & x_{h1} \to 0.425113 \\ x_{h2}^{leader} \to 1.02749 & x_{h2}^{follower} \to 0.256873 & x_{f1} \to 1.12237 & x_2 \to 1.71116 \\ x_{h2} \to 1.28437 & x_{f2} \to 0.426795 & l_{h1} \to 12.7534 & l_{f1} \to 35.9158 \\ l_{f2} \to 14.0842 & l_{h2} \to 37.2466 & w_h \to 0.0322353 & w_f \to 0.0285321 \\ \lambda \to 64.5252 & \Pi_1 \to 0.107618 & \Pi_2 \to 0.0607453 & \Pi_{h1} \to 0.0120039 \\ \Pi_{h2}^{leader} \to 0.0394043 & \Pi_{h2}^{follower} \to 0.00922606 & \Pi_{h2} \to 0.0486303 & \Pi_{f1} \to 0.0956137 \\ \Pi_{f2} \to 0.012115 & \Pi \to 0.168363 & p_1 \to 1 & \end{pmatrix}$$

Cournot competition in sector one and Stackelberg competition with two leaders from h in sector two:

$$\begin{pmatrix} \Psi \to 3.21094 & p_2 \to 0.971276 & x_1 \to 1.5374 & x_{h1} \to 0.392782 \\ x_{f1} \to 1.14462 & x_2 \to 1.72303 & x_{h2} \to 1.31781 & x_{f2} \to 0.405218 \\ l_{h1} \to 11.7834 & l_{f1} \to 36.6278 & l_{f2} \to 13.3722 & l_{h2} \to 38.2166 \\ w_h \to 0.0323204 & w_f \to 0.0284826 & \lambda \to 64.626 & \Pi_1 \to 0.1093 \\ \Pi_2 \to 0.0524906 & \Pi_{h1} \to 0.00993618 & \Pi_{h2} \to 0.0417866 & \Pi_{f1} \to 0.0993642 \\ \Pi_{f2} \to 0.010704 & \Pi \to 0.161791 & p_1 \to 1 & \end{pmatrix}$$

The very first endogenous variable, Ψ is bigger in the Cournot case than in the Stackelberg cases. That means in this setting, the *switch in competition has a negative effect on income.* Using Ricardo's approach to check (in price terms), whether the switch brought a net increase or decrease of output:[31]

$$
\begin{aligned}
x_1^C + p_2^C x_2^C - x_1^{S1} - p_2^{S1} x_2^{S1} &= \\
1.61945 + 0.998908 * 1.62642 - 1.54748 - 0.974633 * 1.71116 &= \\
= 0.028860945 & \qquad (4.85)
\end{aligned}
$$

Equation (4.85) shows that in terms of value the Cournot case delivers a higher output than the Stackelberg case. It is just an other way of calculating from Ψ^{S1} to Ψ^C since 0.028860945 is exactly that difference. Figure 4.12 shows that for the feasible area of Ψ, the Cournot income with given exogenous parameters is always greater than the Stackelberg case.

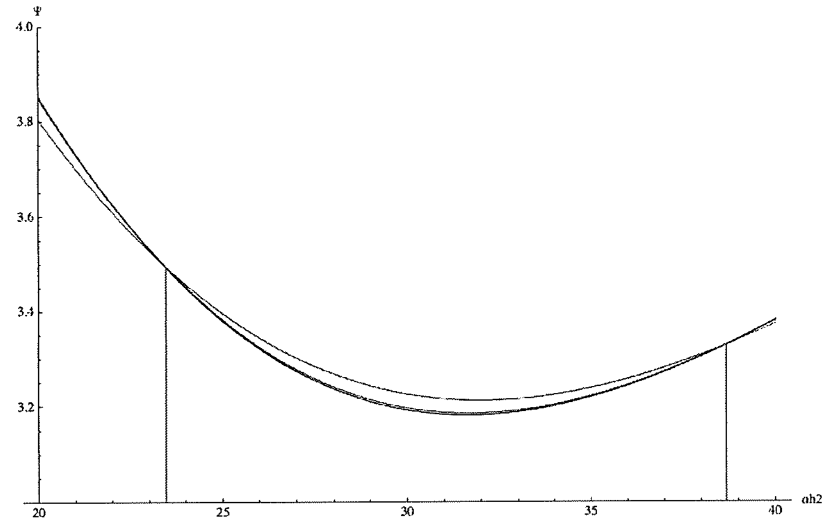

Figure 4.12: Equilibrium world income Ψ of Cournot competition (red), Stackelberg with one leader (purple) and two leaders (blue) for feasible α_{h2} (shaded) at$\alpha_{h1} = 30, \alpha_{f1} = 32$ and $\alpha_{f2} = 33$.

What are the possible channels to decrease total income in the Stackelberg case? The calculation from (4.85) shows the loss from the production side, but this does

[31]The superindices *C*, *S 1* and *S 2* refer to Cournot competition, Stackelberg leadership in industry two with *one* or *two* leaders and three or two followers, one or none in h and two in f repsectively.

not allow a further decomposition for detailed analysis because here have the straigt forward picture that a Stackelberg oligopoly has a higher output and lower price than before. What is interesting is what happens on the income (demand) side of that same economy. Table 4.10 shows the effects on the demand oriented endogenous variables.[32] Looking at table 4.10 and searching for a path to describe the loss in Ψ only allows

Table 4.10: Effect on endogenous variables due to switch in competition

Cournot $\rightarrow$	Stackelberg 1 leader $\rightarrow$	Stackelberg 2 leaders
p_1	$\ominus$	$\ominus$
x_1	$\ominus$	$\ominus$
x_2	$\oplus$	$\oplus$
w_h	$\oplus$	$\oplus$
w_f	$\ominus$	$\ominus$
Π_{h1}	$\ominus$	$\ominus$
Π_{h2}^{leader}	$\ominus$	$\ominus$
$\Pi_{h2}^{follower \rightarrow leader}$	$\ominus$	$\ominus$
Π_{f1}	$\oplus$	$\oplus$
Π_{f2}	$\ominus$	$\ominus$

one answer:

Hypothesis 4. *A move from Cournot to Stackelberg competition with one leader out of four competitors and further on to Stackelberg competition with two out of four being leaders decreases profits in all sectors and all countries except non-leader country*

[32]Mind that the case presented is just a single representative case but the non-linear and rather complex relationship of endogenous variables is simplyfied tremendously by taking parameters for the exogenous or all but one/two exogenous parameters. The author tested the critical relationships of variables and general solution can either be found in the appendix or also requested in Mathematica format. With no further notion, a presented numerical case is representative for the set of feasible allocations, there may though exist non-feasible allocations for which the findings may not hold but these are not subject to research.

profits of the non-leader industry, i.e. Π_{f1}. *Therefore, a Stackelberg case may increase the volume and decrease the price of the product, but higher demand for labor (and through that wages) in the home country and higher profits in non-Stackelberg-industries do not compensate for lower wages abroad, lower profits everywhere in the Stackelberg industry and lower profits in the home non-Stackelberg industry.*

The effects of hypothesis 4 are far-reaching. Sector one at home definitely has lower profits since the higher demand for labor at home drives wages. That also means that workers at home gain from an industry leadership of a home firm (if we leave out the possibility of vertical differentiated multinationals that shift production abroad). Abroad, sector one is also quite well off. The weakened competitors in h allow the producers of good one in f to increase market share, which does not necessarily and actually mean increase sales since demand is lower in the general equilibrium due to the negative income effect of the switch of competition in one industry. Sector two abroad is worse off. They are dictated by one or two leading competitors in h and decrease production below profit maximization. Even the lower wage does not compensate for the lower retail price of the product.

It is very interesting that not only when firm power (leadership knowledge) works against market powers (comparative cost advantages and specialization) world income is decreased, but also when firm power should drive countries to produce more (relatively and absolute) of the good with a comparative cost advantage.

Looking at elasticities and checking what effect income has on utility is another analytical way to look at that problem and interpret the findings. Utility is strictly increasing in income, with $U^C = -3.72308 > U^{S1} = -5.44709 > U^{S2}$. Elasticities show a strictly elastic reaction on the price elasticity of demand. This will be discussed in detail in chapter 5.

4.6.3.2 *Effects on feasible equilibria*

As we see in figure 4.1 and equation (4.84) for one setting of certain exogenous variables, the switch from autarky to trade narrows the set of possible remaining exogenous parameters (in this case α_{h2}). The reason for this is quite obvious: In order to make the comparatively disadvantaged firm survive and not be driven out of the market by the other countries competitors, it must not be too disadvantaged, whereas in the autarky case, there is no other firm type that is better technologically equipped. Also in (4.84), the switch from Cournot to Stackelberg does change feasible values of α_{h2}. Reason for this can be a rise in home wages w_h due to the higher demand for labor since home firms become industry leaders. For all cases of borderline feasible equilibria, the home producer of good one is the lower boundary of positive profits and the upper boundary in α_{h2} on home profits is the home follower in the one leader case and the leaders in the two leaders case.

4.6.3.3 Trade patterns

Consumption can be decompensated if we assume that profits and fixed cost are lump-sum distributed nationally, or:

$$\Psi_h \equiv w_h \Lambda_h + \frac{(\Pi_h + \Pi_f)\Lambda_h}{\Lambda_h + \Lambda_f} + 4F_h, \tag{4.86}$$

$$\Psi_f \equiv w_f \Lambda_f + \frac{(\Pi_h + \Pi_f)\Lambda_h}{\Lambda_h + \Lambda_f} + 4F_f + \zeta G_f \tag{4.87}$$

$$\text{with} \quad \zeta = 0, 1, 2 \text{ for } C, S_1, S_2.$$

Equations (4.86) and (4.88) allow to calculate national incomes and then see what share of world production is consumed in which country. Since the origin is know and preferences are homogeneous internationally, trade patterns are straight forward:[33]

$$\begin{aligned} \Psi_h^C &= 1.69028, \\ \Psi_f^C &= 1.55381, \\ \Psi_h^C/\Psi &= 52.1033\%, \quad \Psi_f^C/\Psi = 47.8967\%, \end{aligned} \tag{4.88}$$

[33]Mind that the superindices d and s refer to demanded (through income) and supplied (through production) quantities of goods one and two.

$$\Psi_h^C/\Psi * x_1 = \quad 0.843786, \qquad (x_{h1}^d) \tag{4.89}$$

$$\Psi_h^C/\Psi * x_2 = \quad 0.847416, \qquad (x_{h2}^d) \tag{4.90}$$

$$\Psi_f^C/\Psi * x_1 = \quad 0.775664, \qquad (x_{f1}^d) \tag{4.91}$$

$$\Psi_f^C/\Psi * x_2 = \quad 0.779. \qquad (x_{f2}^d) \tag{4.92}$$

Equations (4.89) to (4.92) show the national consumption of goods one and two in h and f respectively. Supply happens in the Cournot case as mentioned above in the general equilibrium solution quantity array:

$$\begin{aligned}
x_{h1}^s &= 0.655973, \\
x_{h2}^s &= 1.04555, \\
x_{f1}^s &= 0.963477, \\
x_{f2}^s &= 0.58087.
\end{aligned}$$

Country one thus trades:

$$x_{h1}^s - x_{h1}^d = \quad -0.187813 = \quad 11,5973\% \text{ import of world } x_1 \text{ production,} \tag{4.93}$$

$$13.74436\% \text{ of } x_{h1}^d \text{ are imports,}$$

$$x_{h2}^s - x_{h2}^d = \quad 0.198134 = \quad 12.1822\% \text{ import of world } x_2 \text{ production,} \tag{4.94}$$

$$18.9502\% \text{ of } x_{h2}^s \text{ are exports.}$$

Country two trades:

$$x_{f1}^s - x_{f1}^d = \quad 0.187813 = \quad 19.4933\% \text{ of } x_{f1}^s \text{ are exports,} \tag{4.95}$$

$$x_{f2}^s - x_{f2}^d = \quad -0.198134 = \quad 25.4339\% \text{ of } x_{f2}^d \text{ demand are imports.} \tag{4.96}$$

Equations (4.93) to (4.96) show again that specialized production happens in the Cournot case inline with Ricardo's idea. The following comparison in the Stackelberg 1 and 2 case will show if it also holds for market leadership. Mind that the market leaders operate in a country with a comparative cost advantage, thus Stackelberg should increase trade intuitively. Cases where leadership is in a country with a comparative disadvantage and thresholds will be calculated in a general case (and again α_{h2} as the floating exogenous variable) will be presented in section 5.4.

For one Stackelberg leader and three followers in sector two, income shares times total production (demand as in (4.89) to (4.92)) are:

$$
\begin{aligned}
\Psi_h^{S1} &= 1.7002, \\
\Psi_f^{S1} &= 1.51504, \\
x_{h1}^d &= 0.818298, \\
x_{h2}^d &= 0.904851, \\
x_{f1}^d &= 0.729182, \\
x_{f2}^d &= 0.80631.
\end{aligned}
$$

Supply can again be read in the array of the explicit solution from page 107. In the manner of (4.93) to (4.96), trade patterns are such that:

$$
\begin{aligned}
x_{h1}^s - x_{h1}^d = \; -0.3932 = \; & 23.0003\% \text{ import of world } x_1 \text{ production}, && (4.97) \\
& 48.0492\% \text{ of } x_{h1}^d \text{ are imports}, \\
x_{h2}^s - x_{h2}^d = \; 0.3795 = \; & 22.1788\% \text{ export of world } x_2 \text{ production}, && (4.98) \\
& 41.9422\% \text{ of } x_{h2}^s \text{ are exports.}
\end{aligned}
$$

Country two trades:

$$
\begin{aligned}
x_{f1}^s - x_{f1}^d = \; 0.3932 = \; & 35.0318\% \text{ of } x_{f1}^s \text{ are exports}, && (4.99) \\
x_{f2}^s - x_{f2}^d = \; 0.3795 = \; & 47.0681\% \text{ of } x_{f2}^d \text{ demand are imports.} && (4.100)
\end{aligned}
$$

Comparing these results to Cournot show that Stackelberg competition leads to a higher dependence on foreign production. In the Stackelberg setting though, the higher specialization does not lead to an increase in welfare. This does not contradict Ricardo because specialization does not take place due to cost advantages (and thus productivity and efficiency), but only to power.

For two Stackelberg leaders and two followers, the results are:

$$\begin{aligned}
\Psi_h^{S1} &= 1.70141, \\
\Psi_f^{S1} &= 1.50952, \\
x_{h1}^d &= 0.814639, \\
x_{h2}^d &= 0.913, \\
x_{f1}^d &= 0.722762, \\
x_{f2}^d &= 0.81003.
\end{aligned}$$

$$\begin{aligned}
x_{h1}^s - x_{h1}^d = \quad -0.421857 = \quad & 27.4396\% \text{ import of world } x_1 \text{ production}, && (4.101) \\
& 51.7846\% \text{ of } x_{h1}^d \text{ are imports}, \\
x_{h2}^s - x_{h2}^d = \quad 0.404812 = \quad & 23.4942\% \text{ export of world } x_2 \text{ production}, && (4.102) \\
& 44.3387\% \text{ of } x_{h2}^s \text{ are exports}.
\end{aligned}$$

Country two trades:

$$\begin{aligned}
x_{f1}^s - x_{f1}^d = \quad 0.421857 = \quad & 36.8557\% \text{ of } x_{f1}^s \text{ are exports}, && (4.103) \\
x_{f2}^s - x_{f2}^d = \quad -0.404812 = \quad & 49.975\% \text{ of } x_{f2}^d \text{ demand are imports}. && (4.104)
\end{aligned}$$

For the Stackelberg case with two leaders, specialization towards the Stackelberg leaders intensify in (4.101) and (4.104), in this case it is also in direction of comparative cost advantages.

Concluding, trade patterns follow a sound reasoning. That country with lower (better) technology coefficients is able to generate more than half (at the same country size/endowment) of the world income, see (4.88). In Cournot as well as in Stackelberg competition countries export that good that they have a comparative advantage in, with Stackelberg leading to more specialization than Cournot, but:

Hypothesis 5. *Countries intensify exports of a good with a comparative cost advantage when firms in that country become market leaders, world income though de-*

creases. When firms gain market leadership with a comparative disadvantage, they lower imports and may even switch to exports with sufficient market power. In the latter case, world income drops even more rapidly due to inefficient specialization.

4.6.4 Endogenous Leadership Choice

The essential approach to the question of a net gain in profits for a foreign direct investment *cannot* be made by firms with the evaluation of the whole general equilibrium. Since firms can only gain information on perceived demand and percolated information on income and the foreign cost structure, they will decide on becoming a multinational with fixed endogenous factors, i.e. in partial equilibrium. This leaves room for the result to deviate from the anticipated one since a change in competition changes all endogenous parameters. The question thus is how strong the equilibrium is affected and how high the chance of a firm to have lower profits is than anticipated through FDI. The strategic momentum of own and competitor foreign direct investment will be analyzed from a firm perspective in section 5.4. A home firm that analyses the decision if it should become a multinational faces the problem:

$$\begin{aligned} \pi^C_{h21} = (p_2 - c_{h2})x^C_{h21} - F_h \quad &< \quad \pi^{S1}_{h21} = (p_2 - c_{h2})x^{S1}_{h21} - F_h - G_f, \qquad (4.105) \\ &< \quad \left(\frac{a_2 - b_2 x_2^\dagger}{\lambda^\dagger} - \alpha_{h2} w_h^\dagger \right) x^{S1}_{h21} - F_h - G_f. \end{aligned}$$

$\lambda^\dagger$ and $w_h^\dagger$ indicate that firms calculate with the realized values of the past and cannot take into account the changes in income due to the switch in competition. The presented case fixes all exogenous variables to the values of page 106, the system though is variable in α_{h2}. Also the value of p_2 remains unchanged from the Cournot case and $p_2 = 0.998908$ since the single firm cannot evaluate the whole change in the general equilibrium system. For x_2, the system changes in the distribution of optimal quantities but not in the amount. Why? I a certain period, firms decide to produce $x_2^\dagger$ because of perceived demand. Simultaneous solving of the system by the bypass of Negishi in perceived demand means, that in the next period firms face demand to be $x_2^\dagger$ again if no exogenous parameter changed. Contrary, firms do not realize that they have an influence on demanded quantities except the partial equilibrium knowledge that a lower price will lead to a higher equilibrium quantity, i.e. firms can derive demand except for the income parameters w_f, w_h and Π. Therefore, the firm deciding to undergo FDI and become a market leader can only work with the realized (and therefore demanded) quantity of $x_2^\dagger$ from the Cournot case. As we have seen, the equilibrium quantity of Stackelberg competition is different but this will be discussed later on. Given these parameters of exogenous variables,

$$\begin{pmatrix} b_1 = 10 & b_2 = 10 & a_1 = 80 & a_2 = 80 \\ \Lambda_h = 50 & \Lambda_f = 50 & F_h = 0.001 & F_f = 0.001 \\ \alpha_{h1} = 30 & \alpha_{h2} = 29 & \alpha_{f1} = 32 & \alpha_{f2} = 33 \end{pmatrix}$$

we search for the threshold level of G_f for undergoing foreign direct investment that makes the investment have a positive return on profits in partial analysis. With these results and the Cournot solution of the quadropoly:

$$\begin{aligned} x_{h2} &= 2\frac{a_2 + 2w_f\alpha_{f2}\lambda - 3w_h\alpha_{h2}\lambda}{5b_2}, \\ x_{f2} &= 2\frac{a_2 + 2w_h\alpha_{h2}\lambda - 3w_f\alpha_{f2}\lambda}{5b_2}, \\ &\text{we obtain} \quad x_2^\dagger = 1.62642. \end{aligned} \tag{4.106}$$

Result (4.106) is very important since this is not only the Cournot equilibrium quantity in the general equilibrium case but also the perceived demand for firms in the simultaneous solving process. What the possible market leader knows now is that his investment G_f can change the distribution of $x_2^\dagger$, what remains open is the general equilibrium effect for him. The partial equilibrium part is nothing but implementing Stackelberg competition with the given parameters:

$$x_2^\dagger = x_{h21}^{leader} + x_{h22} + x_{f21} + x_{f22}. \tag{4.107}$$

This changes demand for the profit maximization of the leader to be:

$$
\begin{aligned}
p_2^{leader} &= \frac{a_2 - b_2\left(x_{h21}^{S1} + \frac{a_2 - b_2 x_{h21}^{S1} + 2w_f\alpha_{f2}\lambda - 3w_h\alpha_{h2}\lambda}{4b_2}\right)}{\lambda} \\
&\quad - \frac{b_2\frac{(a_2 - b_2 x_{h21}^{S1} - 2w_f\alpha_{f2}\lambda + w_h\alpha_{h2}\lambda)}{2b_2}}{\lambda}, \quad \text{leading to} && (4.108) \\
\pi_{h21}^{leader} &= \left(p_2^{leader} - \alpha_{h2}w_h^{\dagger}\right)x_{h21}^{leader} - F_h - G_f, && (4.109) \\
\frac{\partial\pi_{h21}^{leader}}{\partial x_{h21}^{leader}} &= 0 \to x_{h21}^{leader} = \frac{a_2 + 2w_f\alpha_{f2}\lambda - 3w_h\alpha_{h2}\lambda}{2b_2}, && (4.110) \\
x_{h22}^{S1} &= \frac{a_2 + 2w_f\alpha_{f2}\lambda - 3w_h\alpha_{h2}\lambda}{8b_2}, && (4.111) \\
x_{f21}^{S1} &= x_{f22}^{S1} = \frac{a_2 - 6w_f\alpha_{f2}\lambda + 5w_h\alpha_{h2}\lambda}{8b_2}. && (4.112)
\end{aligned}
$$

Inserting the values from page 116 for exogenous variables yields:

$$
\begin{aligned}
x_{h21}^{leader} &= 1.30693, && (4.113) \\
x_{h22}^{S1} &= 0.326733, && (4.114) \\
x_{f21}^{S1} &= 0.0943954, && (4.115) \\
x_{f22}^{S1} &= 0.0943954. && (4.116)
\end{aligned}
$$

Equation (4.113) can be substituted into the profit function for the Stackelberg case (4.109). Then, all the information except G_f of (4.105) is collected and the obtained information on G_f is:

$$G_f < 0.024093. \tag{4.117}$$

This means that whenever G_f is smaller than (4.117), the partial equilibrium analysis of firm $h21$ leads to the conclusion that Stackelberg leadership pays off and the firm will invest G_f and undergo FDI for know motives on follower fixed cost information.

Solving the system with leaving α_{h2} open yields for the threshold of G_f the quadratic function:

$$G_f \quad < \quad -0.0452698 + (0.00932939 - 0.000239227\alpha_{h2})\alpha_{h2} \tag{4.118}$$

The corresponding plot is figure 4.13. In this plot, only positive values of G_f make economic sense (otherwise we would have a case of leadership subsidies with Stackelberg competition, a case quite close to (Brander & Spencer 1985)). Additionally, we can see that the higher the comparative cost advantage for the home firm (the lower α_{h2} gets down to $\alpha_{h2} \approx 20$) the higher the threshold can be.

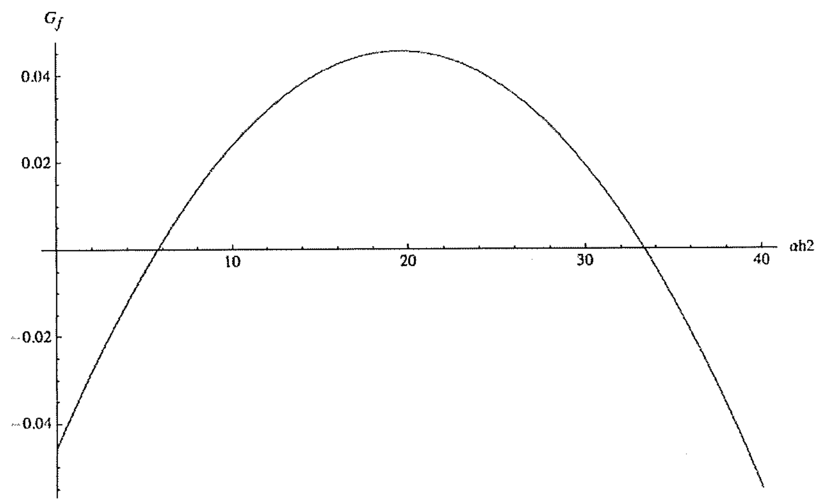

Figure 4.13: Threshold level of additional fixed cost G_f to yield a higher firm profit with Stackelberg leadership.

How big is (in this special case) now the difference between the partial equilibrium anticipation by the firm and the general equilibrium solution? Table 4.11 presents the differences in the price and quantity of the leader, the industry, the wage effects and profits. This is in line with hypothesis 4. We see again a rise in wages and a (not anticipated) drop in profits even below the Cournot level of $\pi_{h21}^{C} = 0.0418319$. The circled plus (minus) indicate a higher (lower) value than expected. This table shows very clearly the importance of general equilibrium analysis for the research subject by the significant differences to an isolated partial equilibrium point of view.

What about the threshold for the second firm to also become a multinational? There

Table 4.11: Difference between partial equilibrium perceived solution and general equilibrium solution

	partial equilibrium *perceived solution*	general equilibrium *equilibrium solution*	
x_{h21}^{leader}	1.30693	1.02749	$\ominus$
x_2	1.82246	1.71116	$\ominus$
p_2^{S1}	0.968184	0.974633	$\oplus$
w_h	0.0316199	0.0322353	$\oplus$
π_{h21}^{leader}	0.0654249	0.0394043	$\ominus$

are two possible scenarios: *First*, both home firms may choose to invest abroad in the same period, which implies a switch from Cournot to Stackelberg competition with two leaders. *Second*, firm $h21$ is already a market leader and the second firm may choose to undergo FDI. This second case has an important strategic momentum: If the second firm has a lower threshold to invest abroad than the first firm had out of Cournot competition, this could be seen as a form of oligopolistic reaction as in (Knickerbocker 1973). Contrary to Knickerbocker though, this would not be due to uncertainty but to a very calculable decision. This is also research subject to chapter 5. It may though end up in uncertainty if we allowed firms to have a parametrization for risk that copes with the differences in their partial equilibrium analysis and the general equilibrium analysis of their decision. In short, this may be an argument for/against the phenomenon of firms undergoing FDI more likely if their home opponents have done so, a case also presented by (Head, Mayer & Ries 2002) in a more analytical framework and (Leahy & Pavelin 2008) in partial equilibrium.

Table 4.12 shows the possible scenarios. The beginning of this section showed scenario a, a move from Cournot to Stackelberg competition with one leader. Two Stackelberg leaders though can be the result of two different stages, either through b directly from Cournot competition or through $a - a$ from Stackelberg competition with one leader. Exactly this latter case of $a - a$ would be oligopolistic reaction from

(Knickerbocker 1973).[34] Game theoretical thoughts on this problem follow after the two cases b and $a-a$ are being presented, where $a-b$ is nothing but the choice of the follower *not* to follow the leader abroad, i.e. no oligopolistic reaction takes place.

Table 4.12: Game tree of foreign direct investment

			$a \nearrow$	oligopolistic reaction - 2 leaders
	$a \nearrow$	1 leader		
			$b \searrow$	first mover advantage - 1 leader
Cournot competition				
	$b \searrow$	2 leaders		

The first case, both firms simultaneously decide to invest G_f abroad to gain market leadership is a move from Cournot competition to a two leader two follower Stackelberg competition. Firms can again only asses the influence on their profits from partial equilibrium analysis, but treat the general equilibrium effects of competition on income and demand as non-existence, i.e. demand is perceived and does not change in its structure. The calculatio of both firms of sector two in h is:

$$
\begin{aligned}
\pi^C_{h2i} = (p_2 - c_{h2})x^C_{h2i} - F_h \quad &< \quad \pi^{S2}_{h2i} = (p_2 - c_{h2})x^{S2}_{h2i} - F_h - G_f, \qquad (4.119)\\
&< \quad \left(\frac{a_2 - b_2 x^\dagger_2}{\lambda^\dagger} - \alpha_{h2} w^\dagger_h\right) x^{S2}_{h2i} - F_h - G_f,\\
&\text{for} \quad i = 1, 2.
\end{aligned}
$$

(4.119) takes again the information from previous periods with no feedback information on wages and income (general equilibrium effects). What firms know is that at the Cournot setting, demand is elastically at given since $\varepsilon_{x_2,p_2} \equiv \frac{\partial x_2}{\partial p_2}\frac{p_2}{x_2} < -1$. This means that firms may find G_f interesting because the firms possibility to raise market output lowers prices but due to the high elasticity will raise revenue. New market output will be:

$$
x^\dagger_2 \quad = \quad x^{leader}_{h21} + x^{leader}_{h22} + x_{f21} + x_{f22}. \qquad (4.120)
$$

[34] Of course, also path b could be oligopolistic reaction in a fitting game setting.

As in section 4.3.1.4, the equilibrium quantities are:

$$
\begin{aligned}
x_{h21}^{S2} &= \frac{a_2+2w_f\alpha_{f2}\lambda-3w_h\alpha_{h2}\lambda}{3b_2} = x_{h22}^{S2}, \\
x_{f21}^{S2} &= \frac{a_2-7w_f\alpha_{f2}\lambda+6w_h\alpha_{h2}\lambda}{9b_2} = x_{f22}^{S2}.
\end{aligned}
$$

With the setting of exogenous variables as usual (page 116), this yields the following values for (partial) equilibrium quantities:

$$x_{h21}^{S2} = x_{h22}^{S2} = 0.871288, \tag{4.121}$$

$$x_{f21}^{S2} = x_{f22}^{S2} = 0.0580917. \tag{4.122}$$

As these equations show, two home leaders can nearly drive their foreign competitors out of the market. What happens when the system of anticipated partial equilibrium movements is calculated in a general equilibrium? Table 4.13 shows the difference between anticipated and realized values.

Table 4.13: Difference between partial equilibrium perceived solution and general equilibrium solution for Cournot to two leader Stackelberg competition

	partial equilibrium *perceived solution*	general equilibrium *equilibrium solution*	
$x_{h2i}^{leaders}$	0.871288	0.658905	$\ominus$
x_2	1.8587594	1.72303	$\ominus$
p_2^{S2}	0.962494	0.971276	$\oplus$
w_h	0.0316199	0.0323204	$\oplus$
π_{h21}^{leader}	0.0418319	0.0223933	$\ominus$

To evaluate whether there exists an *incentive* to undergo FDI of firms think that their national rival will also do so can again be done by leaving α_{h2} open and see how high G_f can be to fulfill (4.119). The solution for this (more general) case is that:[35]

$$G_f \;<\; -0.188852 + (0.0113361 - 0.000170117\alpha_{h2})\alpha_{h2}. \qquad (4.123)$$

The corresponding plot is shown in figure 4.14. Figure 4.14 suggests that *no* level

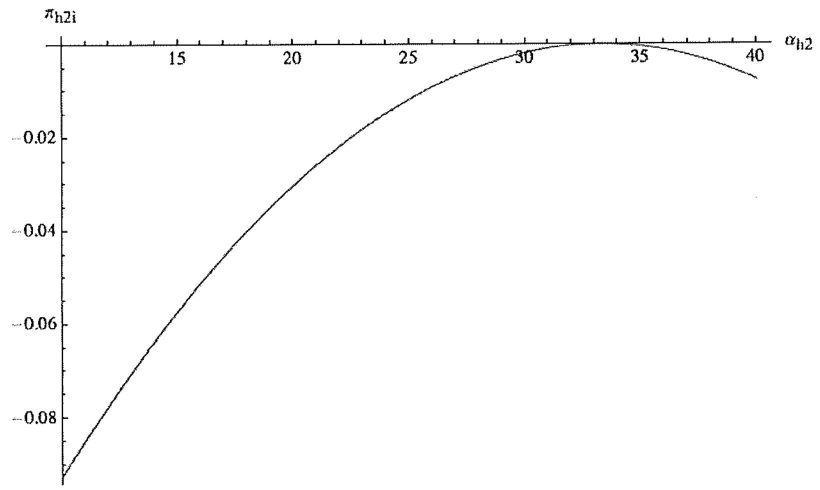

Figure 4.14: Threshold level of additional fixed cost G_f to yield a higher firm profit with Stackelberg 2 leadership out of Cournot competition.

of α_{h2} allows firms to make more profits out of Stackelberg 2 ($S2$) leadership than Cournot.[36] The only level of feasible (non-subsidy or non-negative) $G_f = 0$ can be obtained on the maximum of the threshold function in 4.14, that is at $\alpha_{h2} = 33.3186$. The implication of this is that:

Hypothesis 6. *With positive extra fixed cost of opening a branch abroad (and the know implications on market power), a two leader Stackelberg competition out of Cournot competition is not a dominant alternative for firms.*

[35]At this point it is crucial to remind again that the firms are identical in cost structure and products if they choose the same integration strategy, but their management *cultures* can be different. This does not allow the (too) simplified assumption of firms always acting (fully rationally) in the very same way in the same decision of undergoing FDI or not.

[36]In a partial equilibrium analysis but since we have seen that general equilibrium solutions are lower due to the effect of power concentration there is even less incentive.

Of course, one should not make the mistake to think that this outcome could not be a Nash equilibrium in this non-cooperative game if both firms assess the other firm not to invest G_f and does invest it itself to try to become a single market leader ($S1$).

Game tree $a-a$ in figure 4.12 is Stackelberg 2 ($S2$) leadership out of a single Stackelberg leader situation (oligopolistic reaction). The second-mover has a better (more realistic) baseline of parameters w_h, w_f and λ since firms already know the (realized) general equilibrium outcome of the single Stackelberg leader case ($S1$), which is quite close to the two leader Stackelberg case. The second firm faces the calculatio:[37]

$$\begin{aligned} \pi_{h22} = (p_2 - c_{h2})x_{h22}^{S1} - F_h &< \pi_{h22}^{S2} = (p_2 - c_{h2})x_{h22}^{S2} - F_h - G_f, \qquad (4.124) \\ &< \left(\frac{a_2 - b_2 x_2^\dagger}{\lambda^\dagger} - \alpha_{h2} w_h^\dagger\right) x_{h22}^{S2} - Fh - Gf. \end{aligned}$$

In this case, p_2, w_h and λ are taken from the last realized general equilibrium, the Stackelberg case with one leader. Elasticity at this level is still smaller -1, meaning that firms would expect a rise in revenue due to the higher output, i.e. prices drop slower than quantities rise. Total supply becomes (4.120):

$$x_2^\dagger = x_{h21}^{leader} + x_{h22}^{leader} + x_{f21} + x_{f22}.$$

Firms $x_h 22$ knows that by becoming a multinational (with partial equilibrium exogenous parameters) is a case where it would change its equilibrium (leader) quantity to:

$$x_{h22}^{S2} = \frac{a_2 + 2w_f\alpha_{f2}\lambda - 3w_h\alpha_{h2}\lambda}{3b_2}, \qquad (4.125)$$

$$\text{out of } x_{h22}^{S1} = \frac{a_2 + 2w_f\alpha_{f2}\lambda - 3w_h\alpha_{h2}\lambda}{8b_2}. \qquad (4.126)$$

Inserting all the information on exogenous parameters except α_{h2} from page 116 and taking wages and the marginal utility of income from the one leader case yields the quadratic function of:

$$G_f < 1.34746 + (-0.0834516 + 0.00129209\alpha_{h2})\alpha_{h2}. \qquad (4.127)$$

[37]The leader x_{h21} is already locked in in the leader position, G_f cannot be abandoned.

Equation (4.127) is plotted in figure 4.15: This result is highly interesting. Except for

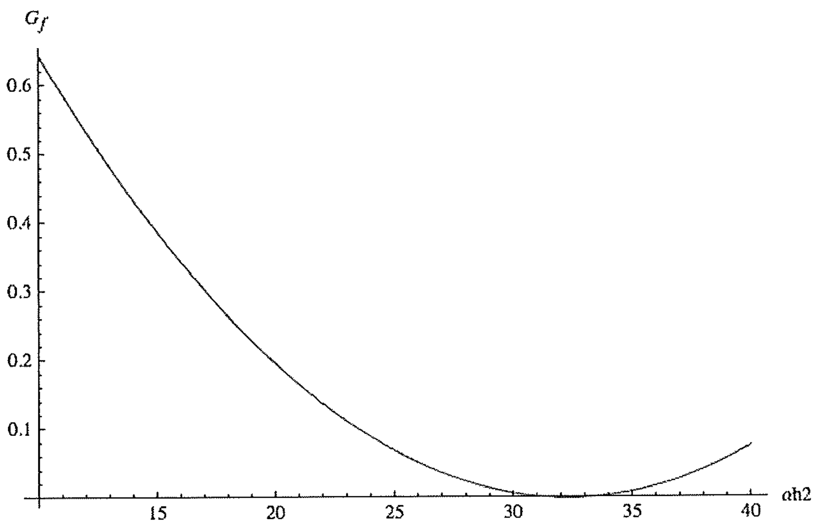

Figure 4.15: Threshold level of additional fixed cost G_f to yield a higher firm profit with Stackelberg 2 leadership out of Cournot competition for the second leader x_{h22}.

one level of $\alpha_{h2} = 32.2933$ where G_f has to be zero to make the second firm become a Stackelberg leader, positive G_f lead to a switch in the type of competition, i.e. firm two decides to invest G_f. The higher the comparative advantage in good two, but also the the higher the disadvantage, therefore *the higher the difference* in relative cost, the higher the possible price to become a leader for the second home firm.

This is very different to the other approach on Stackelberg competition with two leaders where there existed no incentive to do so out of Cournot competition. This game path of staged foreign direct investment in a sense of *follow my leader behavior* as in (Leahy & Pavelin 2003) can be used as an explanation for Knickerbockers oligopolistic reaction, now even in a case without uncertainty.[38]

Hypothesis 7. *A home firm has no incentive to become a multinational (industry leader) if the other home firm does so in the very same period - a move from Cournot to two leader Stackelberg leadership. This changes for the next period, when there is a*

[38] If we assume firms do not realize at all that their decisions will change wages etc. for the whole system, what is implied be the approach in line with (Negishi 1961).

rather high incentive for that firm to follow the leader abroad and also invest G_f. This oligopolistic reaction is not the result of uncertainty but pure cost analysis (in partial equilibrium).

Table 4.14: Difference between partial equilibrium perceived solution and general equilibrium solution for one leader to two leader Stackelberg competition

	partial equilibrium *perceived solution*	general equilibrium *equilibrium solution*	
x_{h22}^{S2}	0.684995	0.658905	$\ominus$
x_2	1.7397	1.72303	$\ominus$
p_2^{S2}	0.97021	0.971276	$\oplus$
w_h	0.0322353	0.0323204	$\oplus$
π_{h21}^{leader}	0.0227396	0.0223933	$\ominus$

Table 4.14 shows again the assumed results and the general equilibrium outcome. The move of the second home firm to become a leader hurts the solely leader of the previous period(s), but this strategic interaction is subject to the following chapter.

4.7 A Condensed Version of the Three General Equilibria

Concluding the core chapter of the general equilibrium I will present a condensed form of all three general equilibria Cournot, Stackelberg competition with one leader and Stackelberg competition with two leaders. The idea is to cut down the rather big general equilibria to a form which contain nine, ten and nine endogenous variables in the same amount of independent equations respectively. A detailed reduction of the models - as this one is - has no dependent equations in the system. We want to solve for the core variables quantities, wages and profits.

The system of the Cournot case is:

$$\left(\begin{array}{c}
x_{\mathrm{h1}} = 2 * \frac{a_1 + 2w_f\alpha_{\mathrm{f1}}\lambda - 3w_h\alpha_{\mathrm{h1}}\lambda}{5b_1} \\
x_{\mathrm{f1}} = 2 * \frac{a_1 - 3w_f\alpha_{\mathrm{f1}}\lambda + 2w_h\alpha_{\mathrm{h1}}\lambda}{5b_1} \\
x_{\mathrm{h1}} + x_{\mathrm{f1}} = \frac{a_1 - p_1\lambda}{b_1} \\
x_{\mathrm{h2}} = 2 * \frac{a_2 + 2w_f\alpha_{\mathrm{f2}}\lambda - 3w_h\alpha_{\mathrm{h2}}\lambda}{5b_2} \\
x_{\mathrm{f2}} = 2 * \frac{a_2 - 3w_f\alpha_{\mathrm{f2}}\lambda + 2w_h\alpha_{\mathrm{h2}}\lambda}{5b_2} \\
x_{\mathrm{h2}} + x_{\mathrm{f2}} = \frac{a_2 - p_2\lambda}{b_2} \\
p_1 * (x_{\mathrm{h1}} + x_{\mathrm{f1}}) + p_2 * (x_{\mathrm{h2}} + x_{\mathrm{f2}}) = w_h * \Lambda_h + w_f * \Lambda_f + \Pi \\
\Lambda_h = \alpha_{\mathrm{h1}} * x_{\mathrm{h1}} + \alpha_{\mathrm{h2}} * x_{\mathrm{h2}} \\
\Lambda_f = \alpha_{\mathrm{f1}} * x_{\mathrm{f1}} + \alpha_{\mathrm{f2}} * x_{\mathrm{f2}} \\
\{p_2, x_{\mathrm{h1}}, x_{\mathrm{f1}}, x_{\mathrm{h2}}, x_{\mathrm{f2}}, w_h, w_f, \Pi, \lambda\}
\end{array}\right)$$

In the Stackelberg case with one leader:

$$\left(\begin{array}{c}
x_{\mathrm{h2l}} = \frac{a_2 + 2w_f\alpha_{\mathrm{f2}}\lambda - 3w_h\alpha_{\mathrm{h2}}\lambda}{2b_2} \\
x_{\mathrm{h2f}} = \frac{a_2 + 2w_f\alpha_{\mathrm{f2}}\lambda - 3w_h\alpha_{\mathrm{h2}}\lambda}{8b_2} \\
x_{\mathrm{f2}} = 2 * \frac{a_2 - 6w_f\alpha_{\mathrm{f2}}\lambda + 5w_h\alpha_{\mathrm{h2}}\lambda}{8b_2} \\
x_{\mathrm{h2l}} + x_{\mathrm{h2f}} + x_{\mathrm{f2}} = \frac{a_2 - p_2\lambda}{b_2} \\
x_{\mathrm{h1}} = 2 * \frac{a_1 + 2w_f\alpha_{\mathrm{f1}}\lambda - 3w_h\alpha_{\mathrm{h1}}\lambda}{5b_1} \\
x_{\mathrm{f1}} = 2 * \frac{a_1 - 3w_f\alpha_{\mathrm{f1}}\lambda + 2w_h\alpha_{\mathrm{h1}}\lambda}{5b_1} \\
x_{\mathrm{h1}} + x_{\mathrm{f1}} = \frac{a_1 - p_1\lambda}{b_1} \\
p_1 * (x_{\mathrm{h1}} + x_{\mathrm{f1}}) + p_2 * (x_{\mathrm{h2l}} + x_{\mathrm{h2f}} + x_{\mathrm{f2}}) = w_h * \Lambda_h + w_f * \Lambda_f + \Pi \\
\Lambda_h = \alpha_{\mathrm{h2}} * (x_{\mathrm{h2l}} + x_{\mathrm{h2f}}) + \alpha_{\mathrm{h1}} * x_{\mathrm{h1}} \\
\Lambda_f = \alpha_{\mathrm{f1}} * x_{\mathrm{f1}} + \alpha_{\mathrm{f2}} * x_{\mathrm{f2}} \\
\{p_2, x_{\mathrm{h1}}, x_{\mathrm{f1}}, x_{\mathrm{h2l}}, x_{\mathrm{h2f}}x_{\mathrm{f2}}, w_h, w_f, \Pi, \lambda\}
\end{array}\right)$$

In the Stackelberg case with two leaders:

$$\left(\begin{array}{c}
x_{h1} = 2 * \frac{a_1+2w_f\alpha_{f1}\lambda-3w_h\alpha_{h1}\lambda}{5b_1} \\
x_{f1} = 2 * \frac{a_1-3w_f\alpha_{f1}\lambda+2w_h\alpha_{h1}\lambda}{5b_1} \\
x_{h1} + x_{f1} = \frac{a_1-p_1\lambda}{b_1} \\
x_{h2} = 2 * \frac{a_2+2w_f\alpha_{f2}\lambda-3w_h\alpha_{h2}\lambda}{3b_2} \\
x_{f2} = 2 * \frac{a_2-7w_f\alpha_{f2}\lambda+6w_h\alpha_{h2}\lambda}{9b_2} \\
x_{h2} + x_{f2} = \frac{a_2-p_2\lambda}{b_2} \\
p_1 * (x_{h1} + x_{f1}) + p_2 * (x_{h2} + x_{f2}) = w_h * \Lambda_h + w_f * \Lambda_f + \Pi \\
\Lambda_h = \alpha_{h1} * x_{h1} + \alpha_{h2} * x_{h2} \\
\Lambda_f = \alpha_{f1} * x_{f1} + \alpha_{f2} * x_{f2} \\
\{p_2, x_{h1}, x_{f1}, x_{h2}, x_{f2}, w_h, w_f, \Pi, \lambda\}
\end{array}\right)$$

With the solution of simultaneous solving for the equilibrium quantities respectively for the three cases:[39]

[39]The solution to the other endogenous variables are rather bulky and can be found in Appendix 2.3.1.

Cournot

$$\begin{pmatrix} x_{\mathrm{h1}} \to \frac{2\alpha_{\mathrm{h2}}\left(-a_2\left(b_2\alpha_{\mathrm{h1}}\alpha_{\mathrm{f1}}^2+b_1\alpha_{\mathrm{f2}}(3\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}}-2\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}})\right)+a_1\left(b_1\alpha_{\mathrm{h2}}\alpha_{\mathrm{f2}}^2+b_2\alpha_{\mathrm{f1}}(3\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}}-2\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}})\right)+3b_1b_2(\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}}-\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}})\Lambda_f\right)+b_2\left(5b_2\alpha_{\mathrm{h1}}\alpha_{\mathrm{f1}}^2+b_1\alpha_{\mathrm{f2}}(9\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}}-4\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}})\right)\Lambda_h}{5b_2^2\alpha_{\mathrm{f1}}^2\alpha_{\mathrm{h1}}^2+5b_1^2\alpha_{\mathrm{f2}}^2\alpha_{\mathrm{h2}}^2+b_1b_2\left(9\alpha_{\mathrm{f2}}^2\alpha_{\mathrm{h1}}^2-8\alpha_{\mathrm{f1}}\alpha_{\mathrm{f2}}\alpha_{\mathrm{h2}}\alpha_{\mathrm{h1}}+9\alpha_{\mathrm{f1}}^2\alpha_{\mathrm{h2}}^2\right)} \\ x_{\mathrm{f1}} \to \frac{2a_2\alpha_{\mathrm{f2}}\left(b_1\alpha_{\mathrm{h2}}(2\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}}-3\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}})-b_2\alpha_{\mathrm{f1}}\alpha_{\mathrm{h1}}^2\right)+2a_1\alpha_{\mathrm{f2}}\left(b_1\alpha_{\mathrm{f2}}\alpha_{\mathrm{h2}}^2+b_2\alpha_{\mathrm{h1}}(3\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}}-2\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}})\right)+b_2\left(5b_2\alpha_{\mathrm{f1}}\Lambda_f\alpha_{\mathrm{h1}}^2+b_1\left(\alpha_{\mathrm{h2}}(9\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}}-4\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}})\Lambda_f+6\alpha_{\mathrm{f2}}(\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}}-\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}})\Lambda_h\right)\right)}{5b_2^2\alpha_{\mathrm{f1}}^2\alpha_{\mathrm{h1}}^2+5b_1^2\alpha_{\mathrm{f2}}^2\alpha_{\mathrm{h2}}^2+b_1b_2\left(9\alpha_{\mathrm{f2}}^2\alpha_{\mathrm{h1}}^2-8\alpha_{\mathrm{f1}}\alpha_{\mathrm{f2}}\alpha_{\mathrm{h2}}\alpha_{\mathrm{h1}}+9\alpha_{\mathrm{f1}}^2\alpha_{\mathrm{h2}}^2\right)} \\ x_{\mathrm{h2}} \to \frac{2\alpha_{\mathrm{h1}}\left(a_2\left(b_2\alpha_{\mathrm{h1}}\alpha_{\mathrm{f1}}^2+b_1\alpha_{\mathrm{f2}}(3\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}}-2\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}})\right)-a_1\left(b_1\alpha_{\mathrm{h2}}\alpha_{\mathrm{f2}}^2+b_2\alpha_{\mathrm{f1}}(3\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}}-2\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}})\right)+3b_1b_2(\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}}-\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}})\Lambda_f\right)+b_1\left(5b_1\alpha_{\mathrm{h2}}\alpha_{\mathrm{f2}}^2+b_2\alpha_{\mathrm{f1}}(9\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}}-4\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}})\right)\Lambda_h}{5b_2^2\alpha_{\mathrm{f1}}^2\alpha_{\mathrm{h1}}^2+5b_1^2\alpha_{\mathrm{f2}}^2\alpha_{\mathrm{h2}}^2+b_1b_2\left(9\alpha_{\mathrm{f2}}^2\alpha_{\mathrm{h1}}^2-8\alpha_{\mathrm{f1}}\alpha_{\mathrm{f2}}\alpha_{\mathrm{h2}}\alpha_{\mathrm{h1}}+9\alpha_{\mathrm{f1}}^2\alpha_{\mathrm{h2}}^2\right)} \\ x_{\mathrm{f2}} \to \frac{2a_1\alpha_{\mathrm{f1}}\left(b_2\alpha_{\mathrm{h1}}(2\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}}-3\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}})-b_1\alpha_{\mathrm{f2}}\alpha_{\mathrm{h2}}^2\right)+2a_2\alpha_{\mathrm{f1}}\left(b_2\alpha_{\mathrm{f1}}\alpha_{\mathrm{h1}}^2+b_1\alpha_{\mathrm{h2}}(3\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}}-2\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}})\right)+b_1\left(5b_1\alpha_{\mathrm{f2}}\Lambda_f\alpha_{\mathrm{h2}}^2+b_2\left(\alpha_{\mathrm{h1}}(9\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}}-4\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}})\Lambda_f+6\alpha_{\mathrm{f1}}(\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}}-\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}})\Lambda_h\right)\right)}{5b_2^2\alpha_{\mathrm{f1}}^2\alpha_{\mathrm{h1}}^2+5b_1^2\alpha_{\mathrm{f2}}^2\alpha_{\mathrm{h2}}^2+b_1b_2\left(9\alpha_{\mathrm{f2}}^2\alpha_{\mathrm{h1}}^2-8\alpha_{\mathrm{f1}}\alpha_{\mathrm{f2}}\alpha_{\mathrm{h2}}\alpha_{\mathrm{h1}}+9\alpha_{\mathrm{f1}}^2\alpha_{\mathrm{h2}}^2\right)} \end{pmatrix}$$

Stackelberg with one leader

$$\begin{pmatrix} x_{\mathrm{h2l}} \to \frac{8\alpha_{\mathrm{h1}}\left(a_2\left(b_2\alpha_{\mathrm{h1}}\alpha_{\mathrm{f1}}^2+b_1\alpha_{\mathrm{f2}}(3\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}}-2\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}})\right)-a_1\left(b_1\alpha_{\mathrm{h2}}\alpha_{\mathrm{f2}}^2+b_2\alpha_{\mathrm{f1}}(3\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}}-2\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}})\right)+3b_1b_2(\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}}-\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}})\Lambda_f\right)+4b_1\left(5b_1\alpha_{\mathrm{h2}}\alpha_{\mathrm{f2}}^2+b_2\alpha_{\mathrm{f1}}(9\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}}-4\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}})\right)\Lambda_h}{4b_2\left(4b_2\alpha_{\mathrm{f1}}^2+9b_1\alpha_{\mathrm{f2}}^2\right)\alpha_{\mathrm{h1}}^2-40b_1b_2\alpha_{\mathrm{f1}}\alpha_{\mathrm{f2}}\alpha_{\mathrm{h2}}\alpha_{\mathrm{h1}}+5b_1\left(9b_2\alpha_{\mathrm{f1}}^2+5b_1\alpha_{\mathrm{f2}}^2\right)\alpha_{\mathrm{h2}}^2} \\ x_{\mathrm{h2f}} \to \frac{2\alpha_{\mathrm{h1}}\left(a_2\left(b_2\alpha_{\mathrm{h1}}\alpha_{\mathrm{f1}}^2+b_1\alpha_{\mathrm{f2}}(3\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}}-2\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}})\right)-a_1\left(b_1\alpha_{\mathrm{h2}}\alpha_{\mathrm{f2}}^2+b_2\alpha_{\mathrm{f1}}(3\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}}-2\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}})\right)+3b_1b_2(\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}}-\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}})\Lambda_f\right)+b_1\left(5b_1\alpha_{\mathrm{h2}}\alpha_{\mathrm{f2}}^2+b_2\alpha_{\mathrm{f1}}(9\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}}-4\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}})\right)\Lambda_h}{4b_2\left(4b_2\alpha_{\mathrm{f1}}^2+9b_1\alpha_{\mathrm{f2}}^2\right)\alpha_{\mathrm{h1}}^2-40b_1b_2\alpha_{\mathrm{f1}}\alpha_{\mathrm{f2}}\alpha_{\mathrm{h2}}\alpha_{\mathrm{h1}}+5b_1\left(9b_2\alpha_{\mathrm{f1}}^2+5b_1\alpha_{\mathrm{f2}}^2\right)\alpha_{\mathrm{h2}}^2} \\ x_{\mathrm{h1}} \to \frac{10\alpha_{\mathrm{h2}}\left(-a_2\left(b_2\alpha_{\mathrm{h1}}\alpha_{\mathrm{f1}}^2+b_1\alpha_{\mathrm{f2}}(3\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}}-2\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}})\right)+a_1\left(b_1\alpha_{\mathrm{h2}}\alpha_{\mathrm{f2}}^2+b_2\alpha_{\mathrm{f1}}(3\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}}-2\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}})\right)+3b_1b_2(\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}}-\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}})\Lambda_f\right)+4b_2\left(4b_2\alpha_{\mathrm{h1}}\alpha_{\mathrm{f1}}^2+b_1\alpha_{\mathrm{f2}}(9\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}}-5\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}})\right)\Lambda_h}{4b_2\left(4b_2\alpha_{\mathrm{f1}}^2+9b_1\alpha_{\mathrm{f2}}^2\right)\alpha_{\mathrm{h1}}^2-40b_1b_2\alpha_{\mathrm{f1}}\alpha_{\mathrm{f2}}\alpha_{\mathrm{h2}}\alpha_{\mathrm{h1}}+5b_1\left(9b_2\alpha_{\mathrm{f1}}^2+5b_1\alpha_{\mathrm{f2}}^2\right)\alpha_{\mathrm{h2}}^2} \\ x_{\mathrm{f1}} \to \frac{2a_1\alpha_{\mathrm{f2}}\left(5b_1\alpha_{\mathrm{f2}}\alpha_{\mathrm{h2}}^2+2b_2\alpha_{\mathrm{h1}}(6\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}}-5\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}})\right)+2a_2\alpha_{\mathrm{f2}}\left(5b_1\alpha_{\mathrm{h2}}(2\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}}-3\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}})-2b_2\alpha_{\mathrm{f1}}\alpha_{\mathrm{h1}}^2\right)+b_2\left(16b_2\alpha_{\mathrm{f1}}\Lambda_f\alpha_{\mathrm{h1}}^2+b_1\left(5\alpha_{\mathrm{h2}}(9\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}}-4\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}})\Lambda_f+6\alpha_{\mathrm{f2}}(5\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}}-4\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}})\Lambda_h\right)\right)}{4b_2\left(4b_2\alpha_{\mathrm{f1}}^2+9b_1\alpha_{\mathrm{f2}}^2\right)\alpha_{\mathrm{h1}}^2-40b_1b_2\alpha_{\mathrm{f1}}\alpha_{\mathrm{f2}}\alpha_{\mathrm{h2}}\alpha_{\mathrm{h1}}+5b_1\left(9b_2\alpha_{\mathrm{f1}}^2+5b_1\alpha_{\mathrm{f2}}^2\right)\alpha_{\mathrm{h2}}^2} \\ x_{\mathrm{f2}} \to \frac{2a_2\alpha_{\mathrm{f1}}\left(2b_2\alpha_{\mathrm{f1}}\alpha_{\mathrm{h1}}^2+5b_1\alpha_{\mathrm{h2}}(3\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}}-2\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}})\right)+2a_1\alpha_{\mathrm{f1}}\left(2b_2\alpha_{\mathrm{h1}}(5\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}}-6\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}})-5b_1\alpha_{\mathrm{f2}}\alpha_{\mathrm{h2}}^2\right)+b_1\left(\left(25b_1\alpha_{\mathrm{f2}}\alpha_{\mathrm{h2}}^2+4b_2\alpha_{\mathrm{h1}}(9\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}}-5\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}})\right)\Lambda_f+6b_2\alpha_{\mathrm{f1}}(4\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}}-5\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}})\Lambda_h\right)}{4b_2\left(4b_2\alpha_{\mathrm{f1}}^2+9b_1\alpha_{\mathrm{f2}}^2\right)\alpha_{\mathrm{h1}}^2-40b_1b_2\alpha_{\mathrm{f1}}\alpha_{\mathrm{f2}}\alpha_{\mathrm{h2}}\alpha_{\mathrm{h1}}+5b_1\left(9b_2\alpha_{\mathrm{f1}}^2+5b_1\alpha_{\mathrm{f2}}^2\right)\alpha_{\mathrm{h2}}^2} \end{pmatrix}$$

Stackelberg with two leaders

$$\begin{pmatrix} x_{\mathrm{h1}} \to \frac{2\alpha_{\mathrm{h2}}\left(-a_2\left(b_2\alpha_{\mathrm{h1}}\alpha_{\mathrm{f1}}^2+b_1\alpha_{\mathrm{f2}}(3\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}}-2\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}})\right)+a_1\left(b_1\alpha_{\mathrm{h2}}\alpha_{\mathrm{f2}}^2+b_2\alpha_{\mathrm{f1}}(3\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}}-2\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}})\right)+3b_1b_2(\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}}-\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}})\Lambda_f\right)+b_2\left(3b_2\alpha_{\mathrm{h1}}\alpha_{\mathrm{f1}}^2+b_1\alpha_{\mathrm{f2}}(7\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}}-4\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}})\right)\Lambda_h}{3b_2^2\alpha_{\mathrm{f1}}^2\alpha_{\mathrm{h1}}^2+5b_1^2\alpha_{\mathrm{f2}}^2\alpha_{\mathrm{h2}}^2+b_1b_2\left(7\alpha_{\mathrm{f2}}^2\alpha_{\mathrm{h1}}^2-8\alpha_{\mathrm{f1}}\alpha_{\mathrm{f2}}\alpha_{\mathrm{h2}}\alpha_{\mathrm{h1}}+9\alpha_{\mathrm{f1}}^2\alpha_{\mathrm{h2}}^2\right)} \\ x_{\mathrm{h2}} \to \frac{2\alpha_{\mathrm{h1}}\left(a_2\left(b_2\alpha_{\mathrm{h1}}\alpha_{\mathrm{f1}}^2+b_1\alpha_{\mathrm{f2}}(3\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}}-2\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}})\right)-a_1\left(b_1\alpha_{\mathrm{h2}}\alpha_{\mathrm{f2}}^2+b_2\alpha_{\mathrm{f1}}(3\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}}-2\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}})\right)+3b_1b_2(\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}}-\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}})\Lambda_f\right)+b_1\left(5b_1\alpha_{\mathrm{h2}}\alpha_{\mathrm{f2}}^2+b_2\alpha_{\mathrm{f1}}(9\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}}-4\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}})\right)\Lambda_h}{3b_2^2\alpha_{\mathrm{f1}}^2\alpha_{\mathrm{h1}}^2+5b_1^2\alpha_{\mathrm{f2}}^2\alpha_{\mathrm{h2}}^2+b_1b_2\left(7\alpha_{\mathrm{f2}}^2\alpha_{\mathrm{h1}}^2-8\alpha_{\mathrm{f1}}\alpha_{\mathrm{f2}}\alpha_{\mathrm{h2}}\alpha_{\mathrm{h1}}+9\alpha_{\mathrm{f1}}^2\alpha_{\mathrm{h2}}^2\right)} \\ x_{\mathrm{f1}} \to \frac{2a_1\alpha_{\mathrm{f2}}\left(3b_1\alpha_{\mathrm{f2}}\alpha_{\mathrm{h2}}^2+b_2\alpha_{\mathrm{h1}}(7\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}}-6\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}})\right)-2a_2\alpha_{\mathrm{f2}}\left(b_2\alpha_{\mathrm{f1}}\alpha_{\mathrm{h1}}^2+3b_1\alpha_{\mathrm{h2}}(3\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}}-2\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}})\right)+b_2\left(9b_2\alpha_{\mathrm{f1}}\Lambda_f\alpha_{\mathrm{h1}}^2+b_1\left(3\alpha_{\mathrm{h2}}(9\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}}-4\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}})\Lambda_f+2\alpha_{\mathrm{f2}}(9\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}}-7\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}})\Lambda_h\right)\right)}{3\left(3b_2^2\alpha_{\mathrm{f1}}^2\alpha_{\mathrm{h1}}^2+5b_1^2\alpha_{\mathrm{f2}}^2\alpha_{\mathrm{h2}}^2+b_1b_2\left(7\alpha_{\mathrm{f2}}^2\alpha_{\mathrm{h1}}^2-8\alpha_{\mathrm{f1}}\alpha_{\mathrm{f2}}\alpha_{\mathrm{h2}}\alpha_{\mathrm{h1}}+9\alpha_{\mathrm{f1}}^2\alpha_{\mathrm{h2}}^2\right)\right)} \\ x_{\mathrm{f2}} \to \frac{2a_2\alpha_{\mathrm{f1}}\left(b_2\alpha_{\mathrm{f1}}\alpha_{\mathrm{h1}}^2+3b_1\alpha_{\mathrm{h2}}(3\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}}-2\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}})\right)+2a_1\alpha_{\mathrm{f1}}\left(b_2\alpha_{\mathrm{h1}}(6\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}}-7\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}})-3b_1\alpha_{\mathrm{f2}}\alpha_{\mathrm{h2}}^2\right)+b_1\left(3\left(5b_1\alpha_{\mathrm{f2}}\alpha_{\mathrm{h2}}^2+b_2\alpha_{\mathrm{h1}}(7\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}}-4\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}})\right)\Lambda_f+2b_2\alpha_{\mathrm{f1}}(7\alpha_{\mathrm{f2}}\alpha_{\mathrm{h1}}-9\alpha_{\mathrm{f1}}\alpha_{\mathrm{h2}})\Lambda_h\right)}{3\left(3b_2^2\alpha_{\mathrm{f1}}^2\alpha_{\mathrm{h1}}^2+5b_1^2\alpha_{\mathrm{f2}}^2\alpha_{\mathrm{h2}}^2+b_1b_2\left(7\alpha_{\mathrm{f2}}^2\alpha_{\mathrm{h1}}^2-8\alpha_{\mathrm{f1}}\alpha_{\mathrm{f2}}\alpha_{\mathrm{h2}}\alpha_{\mathrm{h1}}+9\alpha_{\mathrm{f1}}^2\alpha_{\mathrm{h2}}^2\right)\right)} \end{pmatrix}$$

CHAPTER 5

THE ECONOMICS OF STRATEGY

5.1 UNCERTAINTY AND NON-COOPERATIVE INTERACTION

Section 4.6.4 endogenized the decision of a firm wishing to be an industry leader or not. The two home firms are non-cooperative. When looking at the payoffs (profits of the firms), it is obvious that firms play a prisoner's dilemma game like (Flood 1952).[40] Table 5.1 shows the payoffs in a qualitative form. There are of course also different regions of exogenous parameters, where, with f.e. a sufficiently high G_f there is no incentive to undergo FDI.

Table 5.1: Payoffs in the foreign direct investment game, 1 period, Cournot competition as the status quo

		firm 1 $\neg G_f$	firm 1 G_f
firm 2	$\neg G_f$	$\bigcirc / \bigcirc$ (C)	$\oplus / \ominus\ominus$ (S1)
	G_f	$\ominus\ominus / \oplus$ (S1)	$\ominus / \ominus$ (S2)

As the previous section made clear, being the only leader improves profits personally and weakens the opponents position drastically. Being a multinational alongside the opponent though has no incentive directly out of Cournot (top left directly to bottom right, compare figure 4.14) but has a very high payoff if the other firm is already a

[40]The term prisoners dilemma though was introduced by Albert W. Tucker.

multinational. The latter case is oligopolistic reaction. Obviously, the Pareto efficient setting from a general equilibrium perspective is Cournot competition (C) $(\neg G_f/\neg G_f)$, but the Nash equilibrium turns out to be Stackelberg leadership by both home firms (G_f/G_f) or (S2) since deviating from (C) is too interesting for each of the two firms, leading to (S2) in one or more periods.

The game is not cooperative, else if firms worked together, a one leader solution would lead to highest profits for sector 2 in country h as the results on page 107 suggest. This case of collusion is covered in a different setting of equilibrium (partial and without Stackelberg market leadership) in f.e. (Bernheim & Whinston 1990).

Hypothesis 8. *In the given setting, the Nash equilibrium of both firms undergoing FDI through G_f is not the pareto-efficient allocation (Cournot). In this way, a (tacit) collusion would lead to an intuitively not obvious increased world income.*

Additionally to hypothesis 8, it is important to mention the distribution of income: Wage income rises in the Stackelberg case but not enough to compensate for falling profits. Therefore, workers gain from multinational activity more than shareholders of the corresponding firms.[41]

The payoff matrix when moving from $S1$ leadership by firm 2 to either the same situation or $S2$ and firm 2 also undergoes FDI (oligopolistic reaction) is represented in table 5.2. In this case, the incentive for a firm to follow the leader is *very* high, since

Table 5.2: Payoffs in the foreign direct investment game, 1 period, market leadership firm one as the status quo

		firm 1
		G_f
firm 2	$\neg G_f$	$\bigcirc/\bigcirc$ (S1)
	G_f	$\ominus/\oplus$ (S2)

it does not go abroad as a Cournot oligopolist but as a suppressed follower that can

[41] In the case of the previous chapter, firm profits are distributed in a lump sum fashion, creating shareholders that need not be workers though would not change the system rather than making it more complex with the same results.

switch to a leader status against the foreign competitors. The old leader, firm one, is in a lock in situation since she already has invested G_f and will remain a multinational in any case. Firm two also has to face a significant drop in profits but due to the leader status it is also relatively good of compared to the foreign competitors. This setting clearly is a case of oligopolistic reaction, where the incentive to go abroad is much higher iff the other firm is already a multinational as in (Knickerbocker 1973) and later (Head et al. 2002). Contrary to Knickerbocker though is the determining factor not uncertainty about foreign cost in the first line but the possibility to gain an advantage on the percolating speed of information, which means that uncertainty does not remain for any firm no matter what strategy is chosen, but the speed of obtaining this information (on Ω) changes.

Hypothesis 9. *Oligopolistic reaction incentives, i.e. to follow an other home firm undergoing FDI, are tremendously strong in the given setting, even in the absence of uncertainty (that are known to the firm in the perceived partial equilibrium point of view). Although there is no incentive to reach Stackelberg leadership with two leaders out of Cournot competition (see figure 4.14), the risk of being a follower is high enough to drive firms to invest abroad as they can.*

From a political point of view, it makes no sense to foster foreign direct investment in this setting if general welfare is the goal of interventions. Of course, if the aim is to increase the wages at home, public incentives to invest abroad are favorable since leadership by home firms increases the demand for labor and therefore the wages. We may assume again that labor income recipients are not the shareholders of firms that receive the profits. This is again a question of distribution of wealth that shall not be discussed in detail here. A market solution for the welfare maximizing Cournot setting in sector two is possible if assumptions are such that the *folk theorem* can be applied as in (Rubinstein 1979). The grim trigger, the punishment one firm could threaten in the infinite prisoner's dilemma would be that this firm would follower the deviators strategy to invest abroad. A necessary assumption for this scenario would be that firms can abandon their foreign production, thus both firms return to Cournot and stay with that strategy. At this point of course, the game theoretical outcomes may vary heavily, depending on the assumptions made and may not reflect the decisions made by actual firms since the uprise of foreign direct investment as a strategic and cost tool.

The investment abroad is a strategic complement according to (Bulow et al. 1985). If one firm undergoes FDI, the incentive for the other firm to do so is lower (since the home follower situation is very low in profits and at the margin of economic survival).

5.2 Industrial Organization and Strategy

A typical viewpoint for the strategic momentum of sector two would be to use a *five forces* analysis from (Porter 1979). Table 5.3 illustrates the framework.

Table 5.3: Five forces approach

$$\begin{tikzcd} & \boxed{\text{Entry}} \arrow[d] & \\ \boxed{\text{Supplier Power}} \arrow[r] & \boxed{\text{Internal Rivaly}} & \boxed{\text{Buyer Power}} \arrow[l] \\ & \boxed{\text{Substitutes}} \arrow[u] & \end{tikzcd}$$

The analysis focuses on the strategic possibilities of a certain industry. The idea is to analysis sector two and look for explanations for higher/lower profits of home or foreign, leading or following firms from a business approach. Contrary to the general equilibrium approach presented earlier, this is - due to a very different methodological heritage and point of view - a very dynamic analysis. The literature on performing this analysis besides the original 1979 papers can be found in a more economic language in f.e. (Truett & Truett 2004) or (Besanko, Dranove, Shanley & Schaefer 2007). Discussing the five factors:

Entry is a significant threat for firm under incomplete competition. The whole model of monopolistic competition by (Robinson 1933) and also (Chamberlin 1933) has the number of firms as the endogenous outcome in a general equilibrium setting that is very common in international economics such as in (Markusen 2004). For industrial economics and a focus on strategic interaction though, monopolistic competition does not allow a lot of discussion on strategy and interaction. Models that work with any other approach on oligopolies either keep the number of firms fixed as this model or leave the number of firms open, which is the desired way if one wants to show effects of competition with *one* type of competition such as Cournot in f.e. (Ruffin 2003*b*).

The fear of entry is significant in two ways: *First*, firms working under incomplete competition face more firms dividing the same demand, which is nothing but a lower market share for each single firm. Given the case of fixed cost and linear variable cost, this means higher average cost per unit and lower profits as long as profits are elastic to the price or $\varepsilon_{\pi,p} \equiv \mid \frac{\partial \pi}{\partial p} \frac{p}{\pi} \mid > 1$. *Second*, new firms decrease the power of concentration in an oligopoly leading to a rise in the consumer rent and fall in the producer rent (and deadweight loss), which can be referred to as the positive effect of competition on welfare.

Fixed cost per se decrease the possibility of entry since they represent as a barrier for new firms and protect incumbents. Entry cost marginally higher than zero imply that one has to discard the theory of perfect competition with free entry. One focus of this dissertation is that for research on industrial economics, no sector has ever been found that allows zero entrance cost, thus perfect competition would be a methodologically wrong approach. If these fixed cost were moreover implemented in a model that faces increasing returns to scale, it would be even harder for new firms to enter the market since the returns to scale make it even harder for smaller competitors to survive at given uniform market prices. Also increasing returns to scale without fixed cost would have the same effect.

Other factors that affect entry are possible legal differences, either for all firms of a certain country, such as in (Brander & Krugman 1983) even with Stackelberg competition, or for certain firm within the home sector (national champion). Learning curves, capital knowledge that has to be acquired over time or network externalities are possible but not relevant to this model.

Interaction between firms with respect to previous decisions is covered in the literature on predatory pricing and/or price wars (Bertrand oligopolies), this is definitely an interesting add-on to the given leadership non-cooperative games and could also implement Bertrand (in between all four firms per sector) but would be a model on its own.

This model does not allow entry for simplification reasons, that are non-distoring as (Wilcox 1950) has pointed out. Nevertheless, one should not forget the possibility of entry as an extra feature to the given general equilibrium. A partial equilibrium analysis of Stackelberg leadership with endogenous entry can be found in (Etro 2008), the proof that general equilibrium can solved with n number of firms in a limited number of sectors has been done by f.e. (Ruffin 2003b).

Substitutes are relevant, even though a five forces analysis does not directly investigate demand. At this point, the (usually more theoretical approach) of economics that in this case uses a general equilibrium is more sensitive to demand related issues besides the pure bargaining power of consumers that are being taken care of in the next bullet point of *Buyer Power*.

Substitutes are not available in this model since the interest is in a general equilibrium situation, in which the effects of different forms of competition are more relevant than the fear of substitution of products rise in their prices. Though, if we allowed substitution between goods one and two, the outcome would even strengthen the effects of Stackelberg competition and a further concentration of the market since the Stackelberg sector two rises sales through lower prices, which means that consumers would substitute towards the Stackelberg sector good.

In a certain way, consumers substitute between goods, not very obvious from a strategic management perspective but very ordinary for microeconomics: Consumers maximize their utilities. Therefore, at given market prices, they substitute between goods such that they fulfill the second law of Hermann Heinrich Gossen from (Gossen 1854) if and only if we look at an equilibrium: The marginal utility of each product divided by its price is the same in the general equilibrium, or $\frac{MU_1}{p_1^*} = \frac{MU_2}{p_2^*}$ in equilibrium. This equation is guaranteed by utility maximized demand functions and the simultaneous general equilibrium solution.

Price elasticity of demand or $\varepsilon^C_{x_2,p_2} \equiv \frac{\partial x_2^C}{\partial p_2}\frac{p_2}{x_2^C} = -3.93995$ in the Cournot case[42] is rather high, indicating that a *possible* new substitute good may attract a lot of attention if it is cheaper.

Buyer Power assesses the power of downstream industries or consumers on the sector. It tries to measure the power of consumers in negotiating the price being paid

[42]Elasticity for all three cases of $C, S1, S2$ is always < -3.

and thus increase the consumer rent by lowering the producer rent. In the given model, demand (and thereby buyer power) is very limited in its interaction with the industry since we have introduced *perceived demand* to bypass discussed problems of self-amplifying demand through factor payments. Further on, the power of buyers cannot be significantly high since one assumption is (nearly) perfect competition on factor markets, meaning that there exists a high number of workers and therefore consumers that have no centralized bargaining such as a buyer union. Therefore, buyer power seems to be limited. Also, the number of substitute goods is very limited (to one), meaning that although price elasticity of demand is rather high, the possible channels to flee as a costumer out of sector two are nearly non-existent.

Supplier Power is very low. Perfect competition on the factor market for labor and no unions exist. Factor suppliers do not have a possibility to opt for price discrimination between firms, they are also not mobile internationally. On the other hand, there is one input available and needed only, meaning that in the case of a union, the bargaining power increases tremendously. The model shown could have also been formulated with two inputs, f.e. labor and capital with constant returns to scale, but for the simplicity of Ricardian trade patterns and non-tradeable factors, this has not been chosen.[43]

Internal Rivalry is the attempt and tactics of firms within the sector to increase their market share, not seeing that their decisions on strategies decides about sector-exogenous factors such as wages, income and thus demand. This is definitely the most strategic point of the five forces since it decides whether a firm - is strategic management terms - goes for a quality or price leadership, tries to find niches or controls its rival by size when average cost are decreasing. In this manner, the model is quite clear: The constant returns to scale do not allow to gain advantage through pure size, which is overall a weak argument if we can identify small firms being more profitable than their larger (and maybe more rigid) counterparts in many real sectors. The rather small number of sellers in

[43]Assuming a Cobb-Douglas production function with labor and capital as an input is hard to argue if one has to make the assumption of immobile factors, meaning that capital as an input cannot be transferred internationally. If it was, it would not change the outcome on a qualitative level.

the market suggests that internal rivalry is quite low, but taking a look at the general equilibrium outcomes tells us a different story: There exists quite a large area (of α_{h2} being variable and exogenous), where some firm types can survive and others can not. This means that thinking about the system in a more dynamic way, a change in α_{h2}, f.e. through innovation, may drive out weaker firms. In a different interpretation, the firms being based in a country with a comparative cost advantage (including wages), are technically able to drive their opponents out of the market, in partial equilibrium point of view though. This is because a low cost production would either mean a massive drop in profits or a strong rise in their equilibrium quantity. While the first channel means a drop in income and thus demand, the second channel forces the firms of sector two to get workers from sector one, which is not a viable option. If sector one firms do not set their workers free in order to allow sector two firms to calculate with constant wages, the firms of sector two would face a non-calculable higher wage in the next period (the general equilibrium outcome).

What also heats up competition is that firms can not differentiate between their products, branding is not possible. This means that neither any switching cost within the sector occur for buyers nor that they could have/develop any brand preferences. This also boosts the possibility of firms to undercut their opponents price, which drives the given system towards a Bertrand solution, especially since the price elasticity of demand is high as mentioned before.

The switch to Stackelberg leadership through the informational advantage of percolating fixed cost from abroad would be a typical switch in the strategies of internal rivalry. Firms from country f can only observe this strategy and are assumed not to be able to defect the result. Of course, if they were able to defect, they could signal this and thereby avoid any deviations from Cournot competition, which is a typical setting in the literature on industrial economics and agency theory.

5.3 First and second mover advantage

There clearly exists an incentive for home firms of sector two to invest G_f abroad and become a Stackelberg leader in the given numerical setting. The presented general equilibrium is a one shot simultaneous game, there are no time indices that would allow a first mover (f.e. in period t) and second mover (period $t+1$) calculus. In general, a first-mover advantage would exist if a firm would earn higher profits from going abroad at the very first possibility, or:

$$\pi^{S1}_{(t)\,h2i} > \pi^{C}_{(t)\,h2i}. \tag{5.1}$$

A second mover advantage then would exist iff:

$$\pi^{S2}_{(t+1)h2j} > \pi^{S1}_{(t+1)\,h2i} \wedge \pi^{C}_{(t)\,h2j} + \pi^{S2}_{(t+1)\,h2j} > \pi^{S1}_{(t)\,h2i} + \pi^{S2}_{(t+1)\,h2i}. \tag{5.2}$$

Equation (5.2) contains two parts, the first one is an incentive to go abroad, a participation guarantee. The second part assures that the incentive to go abroad *later* than the opponent is more beneficial overall. If the second part of equation (5.2) can not be fulfilled, but the first part can be, then there clearly exists a first-mover advantage. For the given scenario of one Stackelberg leader,[44] there exists an area of feasible equilibria at the left margin of figure 5.3[45], where home firms of sector one and foreign followers in sector two could be driven out of the market. A systematic analysis of the strategic possibilities of each firm type can be done in a strength-weakness-opportunities-threats (SWOT) analysis:

Home leader x^{S1}_{h21} Given from the general equilibrium plot of figure 5.3, the leader (and follower) can survive an area of comparative advantageous production (low α_{h2}), where the home sector one production is not feasible. This is a very powerful pressure on home competitors on the factor market from the other sector since a shut-down of home good one production strengthens the leaders (and followers) position on factor markets. Compared to the home follower, the leader is able to make good profits even if good two production becomes relatively

[44] This case is chosen since here we have the highest possible number of firm types in the general equilibrium.

[45] This figure has already been presented in the equilibrium description context as figure 4.8.

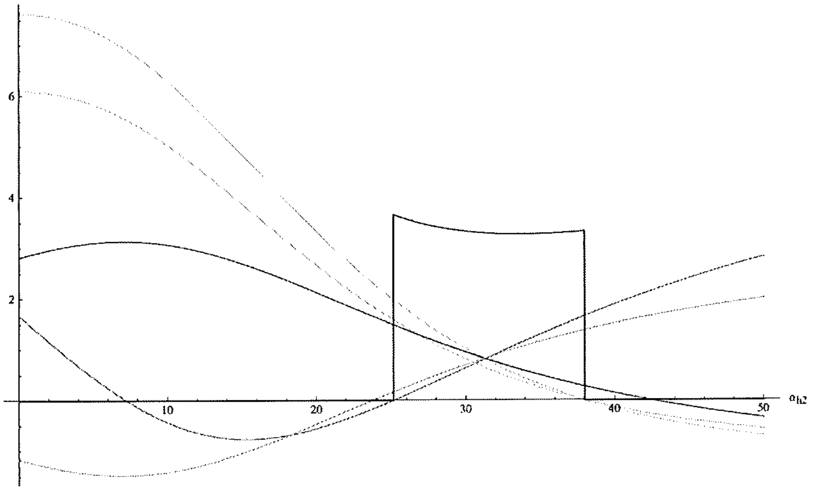

Figure 5.1: Equilibrium quantities of x_{h1} (red), x_{h21}^{leader} (green), $x_{h22}^{follower}$ (yellow), x_{h2} (orange), x_{f1} (blue), $x_{f2}^{followers}$ (brown) and total production $x_1 + x_2 \wedge x_{lj} > 0$ (black/shaded) at $\alpha_{h1} = \alpha_{f1} = \alpha_{f2} = 30$.

expensive. Interestingly enough, leader and follower have to shut down their production at the very same level of α_{h2} (the intersection of the x-axis, the red and green line).

The leader has to invest G_f, which means that he is more dependent on expected sales (from partial equilibrium analysis).

Home follower x_{h22}^{S1} The follower is in a rather weak position but more flexible than the leader since this type can follow (undergo oligopolistic reaction) or not, a situation that may be better if risk aversion on expected and realized values of partial and general equilibria analysis differ significantly. As mentioned above, the profits of the home follower are very dependent on a low α_{h2}.

Foreign followers x_{f2}^{S1} The foreign firms in sector two gain advantage as the relative cost of production good two in the home country rises. They can also survive longer with a decreasing α_{h2} at the mentioned left margin of feasible equilibria.[46] These firms are mostly threatened by foreign direct investment in their

[46]Mind again that *feasible* in this context means that all firm types survive. A situation where certain

country since this strengthens their competitors and weakens world demand (ex post since nobody is able to foresee the effects on profits and wages). An opportunity for these firms may be the possibility to also undergo FDI in the future, which would eliminate all informational time advantages of firms from h and bring Cournot competition again. This benefits in two ways, first a stronger relative position in the own sector and second a risen demand through higher incomes that come with the Cournot solution.

Home sector one x_{h1} Home sector one firms gain strength with a rising comparative cost advantage, this also moves the system towards an equilibrium that makes it harder for sector two home firms to afford G_f (which is decreasing in α_{h2}). They are endangered by two movement: First, a drop in the relative cost of producing good two at home (this indirectly strengthens foreign competitors in sector one through weaker good two producers and this factor demanders abroad. Second, foreign direct investment by sector two firms increases the demand for labor at home and thus factor payments for the same quantity produced.

Foreign sector one x_{f1} Interestingly, foreign production of good one is very enduring concerning the feasible area of α_{h2}. No matter what situation we got on sector two, these two firms make good profits: In a Cournot situation, demand is as high as it can be (except for perfect competition on goods markets), in a one and two leader case, sector 2 competitors from f are weakened, meaning a relatively low wage their country and letting the sector one production in f remain lucrative.The biggest threat for these firms is the mentioned future possibility of their sector two competitors on factor markets in f, because if the sector two in f may become capable of undergoing FDI as the competitors from h, demand and wages rise in f. Still, this Cournot solution on sector two implies higher incomes and thus also a positive income effect partly spent on foreign good one production.

firms (national production of a certain good) drop out of the market have been investigated in the previous chapter and are relevant since they are welfare maximizing to a Ricardian structure and thinkable in global diversification.

5.4 Industry Leadership

The industry leader can set the price earlier than the competitors due to the informational advantage as mentioned in section 4.1.2. This oligopolistic market leadership is intensively discussed in industrial economics and can be found in most textbooks on strategic management since it occurs in multiple industries (Philip Morris in Tobacco, Nestlé in food, Saint Gobain in cement, etc.). Oligopolistic leadership must not be mistaken for a pure barometric leadership in which multiple firms set their prices earlier than competitors, but with a changing move order and just as a reaction to different market condition. Barometric price leadership in this model would be that f.e. foreign firms react faster to changing comparative cost advantages and change their outputs than home firms. Antitrust legislation never aims at barometric price leadership if the price change is in line with exogenous parameter changes, but if it also becomes a signal for other firms to enter some kind of collusion, it is as welfare harming as oligopolistic price leadership.

Price leaders can use different channels to guarantee that their competitors do not undercut their earlier set prices. First, they can introduce a most favored costumer clause that guarantees the costumer to receive the same price for the homogeneous product than from a competitor. As we have seen in the last chapter, this strategy is not necessarily dominant if the leader has higher fixed cost and no increasing returns to scale in the variable cost. In practice, it may be a sufficient deterrence not to enter price wars if all firms live well under the leadership and those who do not may be too weak (in their profits) to challenge the leader. Second, they may be in a market where following firms simply can not differentiate at all (in terms of branding and thus signaling) and just would lower their revenue by selling at a lower price without touching the (set) market price.

The concept of industry leadership can be expanded to a very broad field of research. Research and Development can be implemented as in (Etro 2004), market volume changes and entry as in (Dawid, Kopel & Kort 2010) or (Etro 2008), product differentiation (Zigic 2008) and others. In the given framework, one question on comparative cost structure may be of special interest: How strong is the leadership effect of Stackelberg competition to exactly offset the profit disadvantage through specializing on the comparatively disadvantageous good? We have set the values of

$$\left(\alpha_{h1} = 30 \quad \alpha_{f1} = 32 \quad \alpha_{f2} = 33 \right).$$

For no comparative cost advantage, the ratio of:

$$\frac{\alpha_{f1}}{\alpha_{f2}} = \frac{32}{33} = 0.\bar{96} = \frac{\alpha_{h1}}{\alpha_{h2}}, \tag{5.3}$$

$$\alpha_{h2} = 30.9374\dot{9} \tag{5.4}$$

Equation (5.4) implies that for all $\alpha_{h2} > 30.9374\dot{9}$, country h has a comparative cost disadvantage in good two.

The next step is to calculate the intersection of profits from the industry leader (π_{h21}^{leader}) and one foreign competitors, f.e. π_{f21}, or with the result of the general equilibrium analysis:

$$\pi_{h21}^{leader} = \frac{4385347212978400 - \alpha h2(\alpha h2(\alpha h2(3080439\alpha h2 + 1703755) - 2929846557550) + 226876466638000)}{1000(\alpha h2(1629\alpha h2 - 28160) + 1111760)(\alpha h2(1891\alpha h2 + 33735) + 1460410)} - \tag{5.5}$$

$$\pi_{f21} = \frac{\alpha h2(\alpha h2(\alpha h2(1.89592 \times 10^{6}\alpha h2 - 4.96891 \times 10^{7}) - 8.3199 \times 10^{8}) + 1.5094 \times 10^{10}) + 1.73535 \times 10^{11}}{\alpha h2(\alpha h2(\alpha h2(3.08044 \times 10^{6}\alpha h2 + 1.70376 \times 10^{6}) + 3.53137 \times 10^{9}) - 3.61992 \times 10^{9}) + 1.62363 \times 10^{12}} \tag{5.6}$$

To obtain the maximum possible fixed cost G_f for a plant abroad that allows to make the new leader exactly the same profits as its foreign counterparts, we can set $\pi_{h21}^{leader} = \pi_{f21}$ with two variables open, namely G_f and α_{h2}. Solving this equation for the first one yields:

$$G_f^{equal} = \frac{\alpha h2(\alpha h2((4.96874 \times 10^{7} - 1.899 \times 10^{6}\alpha h2)\alpha h2 + 3.76184 \times 10^{9}) - 2.4197 \times 10^{11}) + 4.21181 \times 10^{12}}{\alpha h2(\alpha h2(\alpha h2(3.08044 \times 10^{6}\alpha h2 + 1.70376 \times 10^{6}) + 3.53137 \times 10^{9}) - 3.61992 \times 10^{9}) + 1.62363 \times 10^{12}} \tag{5.7}$$

The corresponding plot is shown in figure 5.2. As we can see, the values for G_f are rather higher (compared to headquarter fixed cost of $F_h = F_h = 0.001$) with values ranging around 0.1 to 0.3 in the feasible area. The next step is to take the already used parametrization and set $G_f = \frac{1}{2000}$, making a plant abroad being half as expensive as a plant at home. This yields one equation of (5.5) = (5.6) with one unknown, namely α_{h2}:

$$\pi_{h21}^{leader} = \pi_{f21},$$

$$\alpha_{h2} = 31.5181. \tag{5.8}$$

The result of equation (5.8) is the intersection of the red and blue line in figure 5.3. This

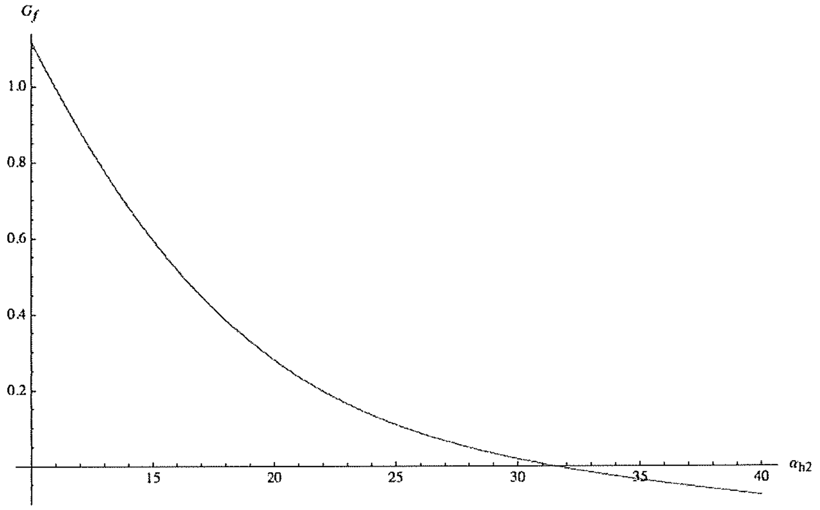

Figure 5.2: Threshold level of G_f to allow the industry leader to receive the same profits as its foreign counterparts

means that in the interval of $\alpha_{h2} \in [30.93749; 31.5181]$, home firms have a comparative disadvantage in good two but the industry leader can use its power at given fixed cost for its extra plant to obtain higher profits than the competitors in f.

What is also remarkable in figure 5.3 is that the follower has minimal positive profits in the comparatively disadvantageous region. Besides the weaker technological possibilities that are represented in a high α_{h2}, this is due to the oppressed relationship with the industry leader and the relatively high wage in the home country since the industry leader has a relatively high market share on the world market. Mind also that profits are highest when α_{h2} is low and just feasible (left top edge of the hyperbolic top cut cylinder). Sector one of foreign firms makes remarkable profits, all over the feasible area, his brown line representing one foreign firm in sector one makes higher profits than the home leader in sector two. This also indicates that the Cournot industry is much more charming for its firms than the Stackelberg industry. In this sense, a possibility to become a multinational is more or less also a burden to enter strategic games that need not improve the absolute situation of a firm, even though the relative position of the leader compared to its followers is a very strong one (even that only applies if we assume this model to be rather static).

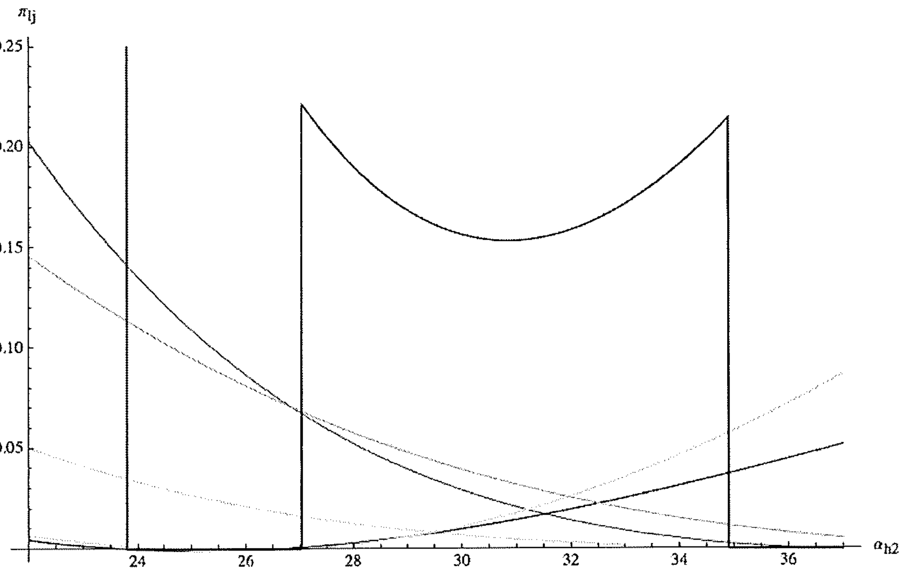

Figure 5.3: Equilibrium profits of π_{h21}^{leader} (red), $\pi_{h22}^{follower}$ (green), π_{f21} (blue), π_{h11} (orange), π_{f11} (brown) and total profits $\pi_1 + \pi_2 \wedge \pi_{lj} > 0$ (black/purple shaded) at $\alpha_{h1} = 30$, $\alpha_{f1} = 32$ and $\alpha_{f2} = 33$.

What can be criticized from a viewpoint of the economics of strategy is the assumption that technology factors are exogenously given, not subject to change (which would represent innovation and learning) or allow any insight on the reasons why certain countries seem to have core competency in some activities. The whole history of a firm or industry, together with the trajectories to possible future change are cut down to one factor that represents which good needs what amount of inputs (time, capital or else) in which country. The central research question of the model in the previous chapter is the influence of a changing *mode* of competition, that - for the sake of a general equilibrium - needs some assumptions on demand and production to guarantee a single equilibrium, else the possible interpretation lacks clarity. Additionally, the assumption of a certain *ceteris paribus*, a static approach in which feedback mechanisms still exist through the general equilibrium approach, is necessary to put light on the black box of a system with more than twenty endogenous variables. What would be changed through using a different formulation of technology, production and cost? Anything but constant returns to scale favors either very large or very small firms but does not change the effect on income of a switch from Cournot to Stackelberg competition. It may change the incentives to undergo FDI in the specific case, but the effects are the same with a high probability, since the given model does not divert from literature on Stackelberg competition with endogenous R&D or changing taste of costumers. From a pure strategy analysis viewpoint, the static technology parameters may even improve the clarity of effects on relative market power, power concentration by a leader and changing incentives to follower or not follower a competitors decision.

5.5 Oligopolistic Reaction

Frederick T. Knickerbocker introduced the term of oligopolistic reaction in his dissertation at Harvard University in (Knickerbocker 1973). The research question was why firms follow their rivals abroad in the very same country, despite - as an empirical work - there were sufficiently other countries that could have been chosen. Knickerbocker proposed that a sufficient level of imperfection of competition, risk aversion and uncertainty make firms follow their opponents. This research inherits one main common feature with this work: Foreign direct investment is neither motivated by serving a foreign market without paying (sufficiently high) transport cost, nor by using a relatively abundant factor from the FDI target country, but for a third and may most delicate reason: Firms match their competitors decision in a strategic interactive play. Oligopolistic reaction is best defined in a formal paper following the everything but brand new idea of Knickerbocker that formalizes his descriptions by (Head et al. 2002): *The decision of one firm to invest overseas raises competing firms' incentives to invest in the same country.* Looking for oligopolistic reaction needs one crucial feature of strategic FDI: The investments need to be strategic complements. Only if one firms' decision to invest the other country (f in the model of this dissertation) raises the incentive for the other firm to do the same (follow), we open the floor to a situation in which one firm initially does not invest abroad but as the home opponents does, it follows. Taking a look at the first section of this chapter, it is pretty obvious that the given model gives reason to act in this way: A firm may not invest in the first period, maybe because it is very prudent and assesses the Cournot equilibrium to be one in which nobody can be better off and is too afraid (risk-avers in a qualitative way) to propose collusion. One competitor is motivated by more than just the qualitative data, more aggressive and maybe less afraid and undergoes FDI. The result of this situation is Stackelberg leadership with one leader and the prudent firm being the follower from home. Of course, this situation makes this firm drastically worse off. The decision to also undergo FDI may be made now, since uncertainty has vanished (the sales of the leading home opponent are tremendous) and the situation got significantly worse, just as described in the previous chapter in 4.6.4 when moving from Stackelberg competition with one leader to the case with two leaders. At this point, all of the firms prudence drops for a clear need to survive the thorny situation of being a home follower with minimal profits, as shown before. The result then is Stackelberg leadership with two leaders, the second home firm has invested in oligopolistic reaction.

The concept of modeling uncertainty about cost with either home or multinational production (or sales without transport cost t) would ask for a modification in the general equilibrium analysis. A short sketch of the answer as given in (Head et al. 2002) would be to let firms formulate their profit functions in the very same way as in the previous chapters and then assume that cost of production at home and abroad are unclear. Utility of firms out of profits are:

$$U_{firms} = -exp^{-\lambda\pi}. \tag{5.9}$$

A formulation of utility for firms as in (5.9) is necessary since firms are not assumed to be risk neutral anymore, but also can be risk averse (or risk friendly). This is represented in λ in (5.9). A firm is then - by definition - risk averse if and only if $\lambda > 0$. With the assumption of normally distributed profits, or $\pi \sim N(\mu, \sigma)$, a monotone transformation as in (Mas-Collel et al. 1995) yields:

$$E_U = \mu_\pi - \frac{\lambda}{2}\sigma_\pi^2. \tag{5.10}$$

The key now is a so called benefit function that measures the expected gain from moving abroad (or opening a branch abroad for sale or in the case of previous chapters obtain an advantage against the home and foreign opponents) versus staying at home, or:

$$B = E_U^{FDI} - E_U^{Home} = \mu_\pi^{FDI} - \mu_\pi^{Home} + \frac{\lambda}{2}\left(\sigma^{2,Home} - \sigma^{2,FDI}\right). \tag{5.11}$$

Equation (5.11) obviously shows that in increase in the risk aversion makes firms move more likely to that country with lower variance in profits. Contrary, a risk neutral firm fully ignores the variance of profits and concentrates on means only. For everything else, and especially a detailed analytical result, it is necessary to measure the share of firms serving the market (or the segregated markets as in (Head et al. 2002) - but this has no significant influence on the result) from abroad by $\zeta = n_f/N$. Then, the benefit function is getting derived partially with respect to x, the share of firms abroad, or:

$$\frac{\partial B}{\partial \zeta} = \left(\frac{\partial \mu_\pi^{FDI}}{\partial \zeta} - \frac{\partial \mu_\pi^{Home}}{\partial \zeta}\right) + \frac{\lambda}{2}\left(\frac{\partial \sigma^{2,Home}}{\partial \zeta} - \frac{\partial \sigma^{2,FDI}}{\partial \zeta}\right). \tag{5.12}$$

The corresponding version for the setting of this dissertation would be:

$$\frac{\partial B}{\partial \zeta} = \left(\frac{\partial \mu_\pi^{S\zeta+1}}{\partial \zeta} - \frac{\partial \mu_\pi^{S\zeta}}{\partial \zeta}\right) + \frac{\lambda}{2}\left(\frac{\partial \sigma^{2,S\zeta}}{\partial \zeta} - \frac{\partial \sigma^{2,\zeta+1}}{\partial \zeta}\right). \tag{5.13}$$

Whenever the derivative of (5.13) and (5.12) are positive, there is oligopolistic reaction since with a rising share of firms operating abroad (and home in (5.13)) increases the incentive to invest in a plant abroad.

With a negative derivative of the benefit function though, FDI is assumed to be a strategic substitute, if the firm is risk neutral. Why? (Head et al. 2002) find that with risk neutral firms, (5.12) is negative. For their setting with segregated markets, this makes perfect sense: Why try to sell the product cheaper to the same costumers abroad like the competitors do, when prices at home, where the not-FDI firm could earn a high producer rent, cannot be cheaper since the competitor moved away and has to pay transport cost to ship products to the initial home country. Put shortly: If one competitor moves its production away, a risk neutral firm has a more positive effect on income at home than substituting production away to the other country (with additional fixed cost).

The assumption of risk averse firms change further reformulation and transformation of (5.12). Without further analysis, the result of (Head et al. 2002) is that with a significant level of risk aversion, a higher variation (σ^2) makes firms follow their opponents since the fear of losing outweighs the risk-neutral argument of higher profits at home.

What is different in this dissertation is that (Head et al. 2002) show oligopolistic reaction with the notion that it occurs only under rather tight assumptions that firms are sufficiently risk averse,[47] we have little competition and foreign cost are uncertain. Little competition is also necessary in the Stackelberg game for obvious reasons but again, industries with *big players* never show signs of perfect competition and cannot fulfill the necessary conditions to be treated as such. Concerning uncertainty, the Stackelberg approach of this dissertation is vague to a certain degree. On the one hand, there is no uncertainty by assumption. On the other hand, there exists an information advantage by the leader compared to its followers. Although this is not uncertainty, there exists informational asymmetry, which - to a certain degree - works in a rather similar. What makes this work similar to Knickerbocker's and the subsequent ideas is that it also shows FDI clustering, which has also been revealed in reality, f.e. by (Kogut & Chang 1991).

[47]In standard agency theory literature, firms are assumed to be risk neutral in most cases, while humans are not. This idea is not a simplification without reasons: Looking at the rise of modern companies, f.e. in the industrial revolution, firms act much more risk neutral and simply dare more as they cannot lose their personal belongings but only company shares. This change in incentives for entrepreneurs changed the way firms acted and contributed a part to the growth of economic output.