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Constitutive models for concrete – Data-driven predictive models for
ultimate lateral shear strength of rectangular and circular reinforced
concrete columns

FINAL REPORT

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General description

This final report describes the obtained results of the research activities from Mohammad Reza Azadi Kakavand during the research visit at University of California, Los Angeles (UCLA) from October 15th 2018 until 15th January 2019. It should be noted that regular Skype meetings were arranged before initiation of my visit at UCLA, which helped a lot to reach the pre-defined goals. Albeit, three-month visit was a short time to achieve publishable results, nonetheless, working hard and valuable guidance by both home and host supervisors resulted in publishable results, which will be discussed in the following.

The main goal of this research visit can be described as developing a novel concrete model, which is able to capture the realistic behavior of concrete under cyclic loading. Hence, this study is consisted of several parts as follows:

First off, different concrete models, which are the most applicable ones in practical engineering calculations, were studied. To evaluate the performance of those models to simulate the behavior of concrete material under various loading conditions, several numerical simulations were performed. The comparison of the obtained results by the numerical simulations with the observed ones during tests revealed the major shortcoming of the existing concrete models. This part of study was completed before beginning of my visit at UCLA. The next steps were proceeded in collaboration with Prof. Taciroglu, which will be discussed in the following sections.

The second step was devoted to investigate the probable solutions to modifying the cyclic behavior of the existing concrete models. In this regard, different mathematical functions together with true physical meaning were considered by means of mechanical properties of concrete material. Next, one of the most popular concrete models was modified by using the modifying function. Finally, the performance of the enhanced model was evaluated by means of several experimental monotonic and cyclic tests such as uniaxial tension and compression tests, biaxial and triaxial compression tests.

Finally, it was observed that there is a very good agreement between the obtained results by numerical simulations with experimental results. Hence, a manuscript was written based on the achieved results, which is almost ready for submission to a major international journal. The publication builds the basis of this report. In the following, the novel model for cyclic behavior of concrete under various types of lateral loading will be presented and discussed.

After writing the manuscript regarding the constitutive models for concrete, which is the main topic of my research visit at UCLA, another research study regarding a novel data driven model for predicting the shear strength capacity of rectangular and circular reinforced concrete columns started. This work can be related to the main topic since the first study investigates the behavior of plain concrete at material level subjected to cyclic loading and the second one assesses the ultimate shear strength capacity of concrete columns reinforced by longitudinal and transverse reinforcements, which is a vital parameter for evaluating the stability and resistance of reinforced concrete structures during lateral citations like earthquakes.

In the second study, two novel empirical equations were proposed and validated by means of two extensive experimental databases. The equations were developed by employing statistical methods (i.e. linear and nonlinear regression analyses). The proposed models can give the practical engineers and researchers an accurate estimation about the lateral strength capacity of two aforementioned types of columns. Consequently, a manuscript is written and ready for submission to an international journal due to very good agreement between the numerical and test results. The proposed models will be discussed at the end of this report.

Chapter 1: Constitutive models for concrete

This chapter is devoted to present the enhanced concrete damage plasticity model and the obtained results for cyclic loading. The comparison of numerical and experimental results will be discussed as well.

Abstract

In the literature, some concrete damage-plasticity models employ one damage parameter for both tension and compressive regimes and some introduces two damage parameters. In the latter models, the damage parameters play their roles independently during cyclic loading, which can be described as the compressive damage does not have any influence on the tensile behavior of concrete or vice versa. This study aims to propose an enhanced concrete model that is able to capture the cyclic behavior of concrete material subjected to multiaxial loading conditions (i.e. uniaxial, biaxial and triaxial) and considers the effect of compressive/tensile damage on tensile/compressive behavior. The proposed Enhanced Concrete Damage Plasticity model, which is called ECDP model, is an update of an existing model, which is developed by combination of the theory of plasticity and damage mechanics theory. Thereby, the performance of ECDP model is evaluated by means of the experimental results under cyclic loading with multiaxial loading conditions. The comparison of the numerical and experimental results demonstrated the capability of ECDP to capture the cyclic behavior of concrete.

Introduction

Several experimental and numerical research activities have been carried out to develop and validate various models for plain and reinforced concrete, aimed to predict the realistic behavior of concrete structures under lateral loadings like earthquakes (Ellingwood 2001; Sezen 2002; Elwood 2004; Zhu and Elwood 2007; Haselton et al. 2008; Yavari 2011; Vii et al. 2012; Adibi et al. 2018; Azadi and Allahvirdizadeh 2018). Meanwhile, some the aforementioned models tried to describe the behavior of structural elements (i.e. beams, columns, etc.) and some others focused on material level (Kent and Park 1971; Popovics 1973; Mander et al. 1988; Scott et al. 1989). These models enable practical engineers and researchers to more accurately predict the behavior of concrete material and consequently the structural response. Constitutive models for concrete, which are solely applicable for monotonic loading, can not describe some realistic characteristics that reinforced concrete elements exhibit under cyclic loading, such as opening/closing of cracks or damage pattern, etc. These constitutive models have been proposed in the literature that aim to predict concrete behavior at material (i.e. plain concrete) or structural levels (i.e. reinforced concrete members) (Willam 1975; Mazar and Pijaudier-Cabot 1989; Pramono and Willam 1989; Etse and Willam 1994; Pivonka 2001; Tao and Phillips 2005; Grassl and Jirásek 2006; Papanikolaou and Kappos 2007). Some of these models can only describe the uniaxial behavior of unconfined concrete, also known as plain concrete, (Hognestad 1951; Kent and Park 1971; Popovics 1973; Thorenfeldt 1987; Tsai 1988) or confined concrete, also known as reinforced concrete, wherein the confinement is provided by stirrups that increases the ultimate compressive strength and ductility capacities of concrete (Roy and Sozen 1965; Kent

and Park 1971; Park et al. 1975; Park and Priestley 1982; Sheikh and Uzumeri 1982; Mander et al. 1988; Yong et al. 1988; Scott et al. 1989; Esmaily and Xiao 2002).

Furthermore, some existing constitutive models have demonstrated their capabilities for simulating the cyclic and dynamic behavior of concrete material. Albeit, some uncertainties about their performance still exist. Some of these models are concisely reviewed in the following and the major shortcoming of the two most applicable concrete damage plasticity models is also discussed next.

One of the primary constitutive models to describe the dynamic behavior of plain concrete was proposed by Bićanić and Zienkiewicz (1983), which follows the theory of elasto-viscoplasticity (Perzyna 1966). In the following, another model was proposed based on damage mechanics theory to capture the biaxial tension-compression behavior of concrete (Suaris and Shah 1985). The impacts of high dynamic loading conditions and large deformation on concrete failure were investigated using a numerical model proposed by Rabczuk and Eibl (2006). Sima et al. (2008) employed two independent damage variables for compressive and tensile regimes to describe strength and stiffness degradation, opening/closing of cracks subjected to uniaxial cyclic loading. Smearred rotation crack approach was employed to propose a constitutive model, which can describe some important characteristics of concrete material under cyclic loading such as opening and closure of cracks, inelastic strains and strength and stiffness degradation (He et al. 2008). Moreover, the Elasto-Plastic-Fracture-Damage (EPFD) theory was also examined to capture the cyclic response of plain and reinforced concrete by means of two independent damage variables for crushing in compression and fracture in tension (Tasnimi and Lavasani 2011). Since the constitutive model by Sima (2008) can not describe damage accumulation in unloading regime, it was modified by Breccolotti et al. (2015). Moharrami and Koutromanos (2016) combined the theory of plasticity and smeared crack model to propose a constitutive model to describe the cyclic behavior of concrete. The cyclic response of concrete was assessed to consider crushing and cracking in compression and tension, respectively, bond stress between concrete and stirrups, and steel yielding at crack-crossing using a constitutive model proposed by (Huguet et al. 2017)

One of the most popular and practical use models for simulating the behavior of plain and reinforced concrete under monotonic, cyclic and dynamic loadings was proposed by Lee and Fenves (1998). The model is developed based on the combination of classical theory of plasticity and continuum damage mechanics theory, which is typically known as the Concrete Damage Plasticity model (henceforth CDP). CDP can describe stiffness degradation due to crushing in compression and cracking in tension, strength recovery caused by opening and closure of cracks and also the impact of lateral confinement on strength and ductility capacities. Grassl et al. proposed a Concrete Damage Plasticity Model (henceforth CDPM2), aimed to describe the load changes from tension regime to compression regime or vice versa, using two independent damage parameters. The model is also able to capture the increase in strength and ductility capacities due to increasing lateral confinement. Despite their aforementioned performance, none of them can not capture the effect of tensile/compressive damage on the compressive/tensile response during cyclic analysis. Hence, this study aims to address this shortcoming through the development of a new model named as Enhanced

Damage Plasticity Model (henceforth, the ECDPM). In the following, the capability of the model to describe the cyclic response of plain concrete under multiaxial loading conditions will be discussed.

A brief introduction of the Enhanced Damage Plasticity Model for concrete

The proposed ECDPM can be introduced as an enhanced version of the concrete damage plasticity model by Grassl and Jirásek (2006), known as CDPM1, which is only applicable for simulating the concrete behavior under monotonic loading since it employs a single damage variable for both compression and tension regimes. The enhanced model can describe some important characteristics of concrete such as isotropic hardening and softening, accumulation of inelastic strains, strength and stiffness degradation caused by compressive or tensile damage. The nominal stress-strain relation for ECDPM reads

$$\boldsymbol{\sigma} = (1 - \omega)\mathbf{E}_0 : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^P) \quad (1)$$

In Eq. (1), $\boldsymbol{\sigma}$ is the nominal stress, ω is the scalar damage parameter ranging from 0 (intact material) to 1 (fully damaged material), \mathbf{E}_0 the elastic stiffness tensor, $\boldsymbol{\varepsilon}$ is the total strain tensor, and $\boldsymbol{\varepsilon}^P$ is the plastic strain tensor. The plastic part of the model can be stated in terms of the effective stress as follows,

$$\bar{\boldsymbol{\sigma}} = \frac{\boldsymbol{\sigma}}{(1 - \omega)}. \quad (2)$$

The model can describe the softening response in post-peak region due to crushing in compression or cracking in tension by employing two damage parameters as follows,

$$(1 - \omega) = (1 - \omega_c)[1 - r(\bar{\boldsymbol{\varepsilon}})\omega_t], \quad (3)$$

where, ω_c and ω_t are the compressive and tensile damage parameters, respectively, $\bar{\boldsymbol{\varepsilon}}$ are the principal strain components, and the strain-based split weight factor of $r(\bar{\boldsymbol{\varepsilon}})$ ranging from 0 (total compression) to 1 (total tension), which takes the form

$$r(\bar{\boldsymbol{\varepsilon}}) = \frac{\sum_{l=1}^3 \langle \bar{\varepsilon}_l \rangle}{\sum_{l=1}^3 |\bar{\varepsilon}_l|}. \quad (4)$$

In Eq. (4), $\langle \bar{\varepsilon}_l \rangle$ are the positive components of the principal strains (Zreid and Kaliske 2018). It is worth noting here that the other damage plasticity models (Lee and Fenves 1998; Grassl et al. 2013) use a stress-based split weight factor, which yields unrealistic response during uniaxial cyclic loading.

Fig. 1 demonstrates the impact of different split weight factors on the cyclic behavior of plain concrete. In this regard, the computed response by ECDPM is compared to the predicted results by two damage plasticity models. It can be followed from Fig. 1 that the tensile damage is occurred ($\omega_t > 0$) causes degradation of tensile modulus of elasticity ($E_t < E_0$). Albeit, since the loading direction changes from tension to compression ($\omega_c = 0$), the value of modulus of elasticity in compressive regime is equal to the initial value ($E_c = E_0$), caused by $r(\bar{\boldsymbol{\sigma}}) = 0$. It should be noted that Grassl et al. (2013) employed a bilinear function to represent the softening response in tension, while an exponential function is used in this study

for the sake of simplicity according to (Grassl and Jirásek 2006). The effect of strain-based split weight factor on the cyclic response of concrete in compressive regime such as stress-strain relation and compressive Young's modulus is illustrated in Fig. 1. It can be followed that the compressive Young's modulus decreases due to the tensile damage. Thus, the use of a strain-based split weight factor is the main improvement over the other damage plasticity models (Lee and Fenves 1998; Grassl et al. 2013).

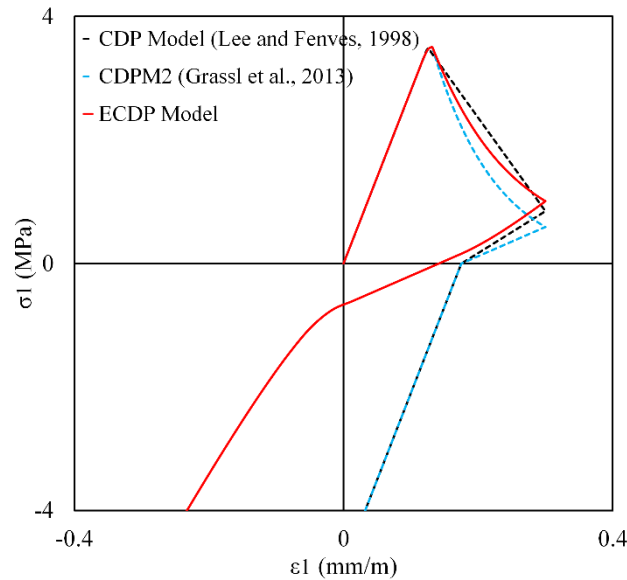


Figure 1 – Comparison of the predicted results between the ECDPM and other damage plasticity models by employment of strain-based and stress-based split weight factors, respectively.

Validation of ECDPM with experimental results

In this section, the computed results by ECDPM is compared to the ones measured during experimental tests. To this aim, several tests subjected to cyclic or monotonic loading patterns, including uniaxial tension and compression tests, biaxial and triaxial compression tests are used. The comparison of results is discussed in the following subsections.

Uniaxial tension and compression tests under monotonic and cyclic loading

First, the capability of ECDPM is evaluated using three uniaxial tension tests under monotonic loading carried out by Li et al. (1998). To simulate these tests, the material parameters are set as Young's modulus $E = 31$ to 38.2 GPa, uniaxial compressive strength $f'_c = 46.2$ to 52.4 MPa, uniaxial tensile strength $f_{tu} = 3.9$ to 4.2 MPa, the Poisson's ratio ν is assumed as 0.2 , the specific mode I fracture energy G_{FI} is estimated as 0.15 N/mm according to the fib model code for concrete structures (Taerwe et al. 2013) and the characteristic length l_{char} is taken as 120 mm, which is equal to the specimens' length. The computed results by the numerical simulations and measured ones from tests are shown in Fig. 2. The comparison of results reveals that the ECDPM captures the ultimate tensile strength and the softening response very well.

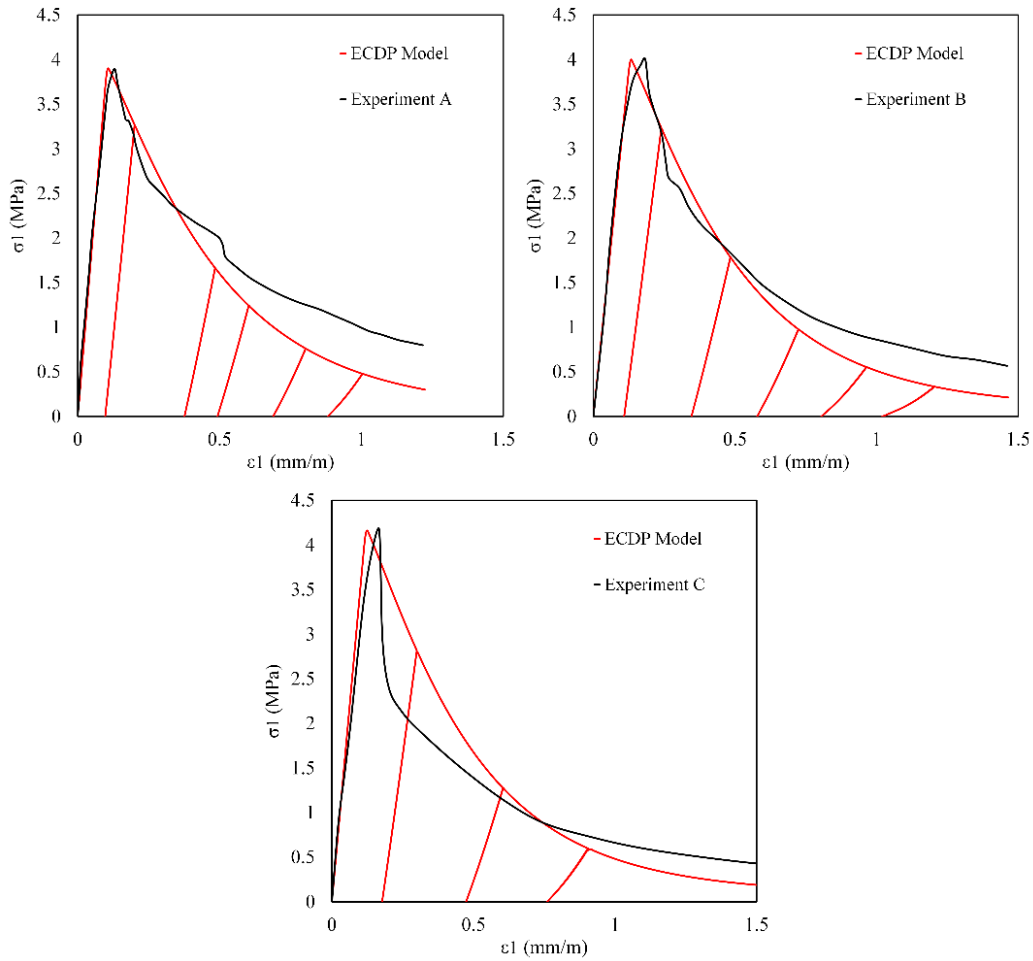


Figure 2 – Comparison of measured and computed cyclic behavior of concrete in uniaxial tension tests under monotonic loading conducted by Li et al. (1998).

It should be noted that the employed characteristic length for simulating the following tests is 100 mm as reported.

In the second example for evaluating the performance of ECDPM to simulate the uniaxial cyclic tension test, the predicted concrete response is compared to the observed one conducted by Gopalaratnam and Shah (1985). The material parameters for the numerical simulation are set as $E = 28$ GPa, $\nu = 0.20$, $f_{cu} = 40$ MPa, $f_{tu} = 3.5$ MPa and G_{FI} is reported as 0.055 N/mm. The comparison of the obtained results versus the measured one is shown in Fig. 3. It can be followed that there is an excellent agreement between the numerical and experimental results.

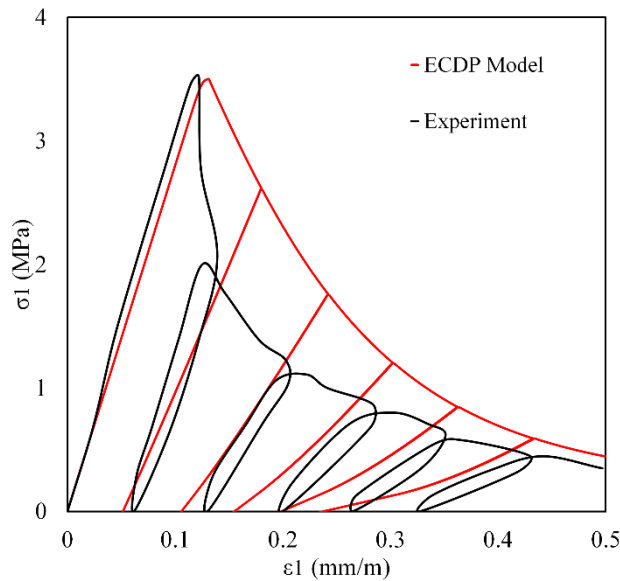


Figure 3 – Comparison of measured and computed cyclic behavior of concrete in uniaxial tension test under cyclic loading conducted by Gopalratnam and Shah (1985).

Next examples in this subsection focus on evaluating the capability of ECDPM to simulate the cyclic response of plain concrete subjected to uniaxial compression loading. In this regard, the computed results by ECDPM are compared to the ones reported by two experimental studies conducted by Karsan and Jirsa (1969) and Van Mier (1984).

The material parameters for the uniaxial compression test carried out by Karsan and Jirsa (1969) are given as $E = 30$ GPa, $\nu = 0.20$, $f_{cu} = 28$ MPa, $f_{tu} = 2.8$ MPa and G_{FI} was computed according to the fib model code for concrete structures as 0.14 N/mm. The comparison of numerical and experimental results is presented in Fig. 4. It can be seen in Fig. 4 that the ECDPM computes the ultimate uniaxial compressive strength and compressive softening response very well.

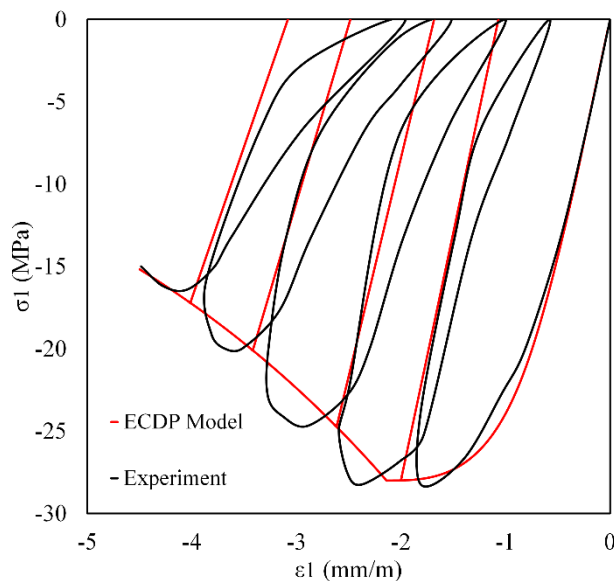


Figure 4 – Comparison of measured and computed cyclic behavior of concrete in uniaxial compression test under cyclic loading tested by Karsan and Jirsa (1969).

The second example for evaluating the performance of ECDPM to predict the concrete behavior under uniaxial compressive loading is conducted by Van Mier (1984). The employed

material parameters for numerical simulation are set as $E = 33$ GPa, $\nu = 0.20$, $f_{cu} = 42.3$ MPa, $f_{tu} = 2.8$ MPa, and $G_{FI} = 0.15$ N/mm. The computed and measured results are displayed in Fig. 5. Fig. 5a demonstrates that the ultimate uniaxial compressive strength and the softening response are predicted very well by ECDPM along direction 1 (loading direction). However, it can be clearly observed that ECDPM predicts the strains in direction 2 and 3 extremely smaller than the measured ones during test. Van Mier reported that the strains at free surfaces always remained smaller than reported values by strain gauges after reaching the peak strength. It was supposed that this considerable discrepancies were caused by surface unloading phenomena (Van Mier 1984).

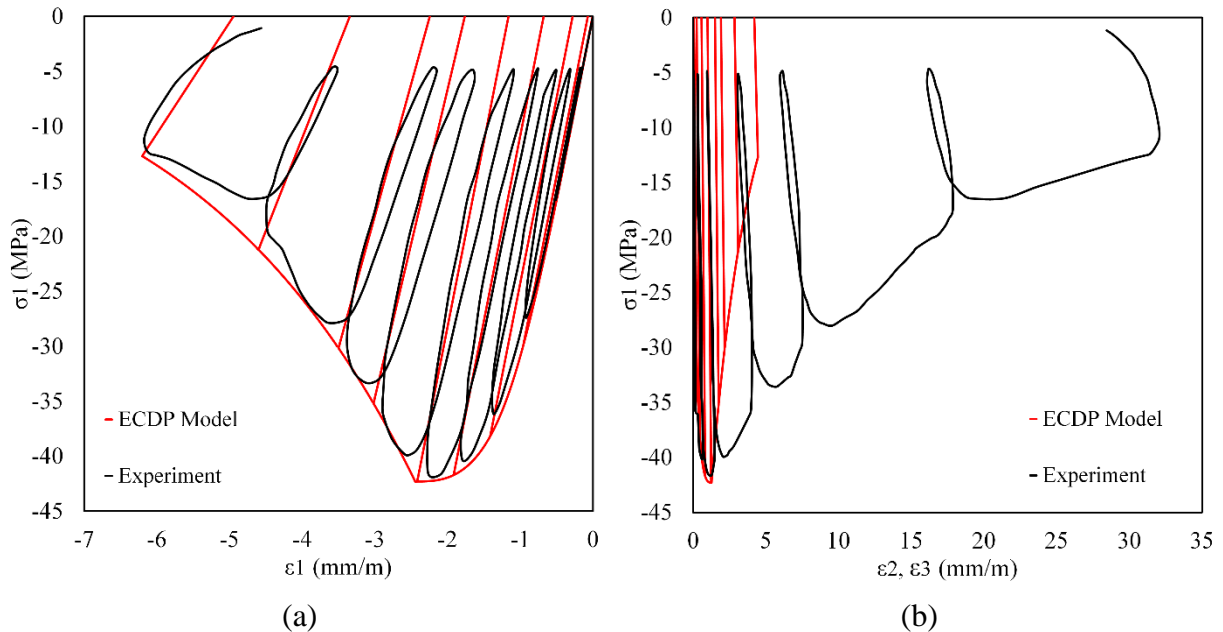


Figure 5 – Comparison of measured and computed cyclic behavior of concrete in uniaxial compression test under cyclic loading tested by Van Mier (1984).

Biaxial compression tests under monotonic and cyclic loadings

This subsection is devoted to evaluate the performance of ECDPM to simulate the behavior of concrete under monotonic and cyclic using two biaxial compressive tests by Kupfer et al. (1969) and Van Mier (1984).

In the first example, the predicted results by ECDPM are compared to the experimental results by Kupfer et al. (1969). In this test, the employed material parameters for numerical simulations are $E = 32$ GPa, $\nu = 0.18$, $f_{cu} = 32.8$ MPa, $f_{tu} = 3.3$ MPa, and the Mode-I fracture energy G_{FI} is estimated as 0.145 N/mm. To perform biaxial compressive tests, the two tested specimens were subjected to the lateral confining pressure of 50% and 100% of the ultimate uniaxial compressive strength of concrete. The computed responses from numerical simulations and the observed ones are presented in Fig. 6. It can be seen in Fig. 6 that the concrete responses in these uniaxial and biaxial tests are captured very well by ECDPM.

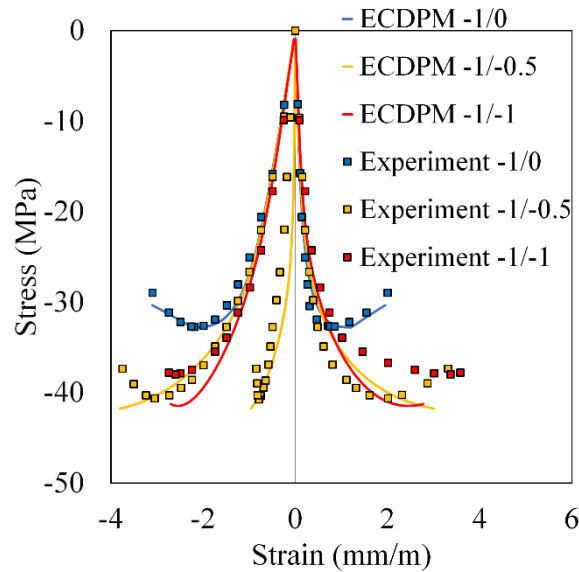


Figure 6 – Comparison of measured and computed cyclic behavior of concrete in uniaxial and biaxial compression test under monotonic loading tested by Kupfer et al. (1969).

The second example compares the obtained results from numerical simulations by ECDPM and the measured ones during biaxial cyclic compression test conducted by Van Mier (1984). The used material parameters are given as $E = 25$ GPa, $\nu = 0.20$, $f_{cu} = 47.9$ MPa, $f_{tu} = 3.1$ MPa, $G_{FI} = 0.15$ N/mm. The specimen was subjected to lateral confinement pressure as 5% of the ultimate uniaxial compressive strength of concrete. Fig. 7a demonstrates the capability of ECDPM to predict the ultimate compressive strength of concrete and the softening response along the loading direction and also the directions of confining pressure. Moreover, it can be seen that the influence of confinement on the ultimate compressive strength is captured very well by ECDPM. However, like the obtained results for the numerical simulation of uniaxial compression test by Van Mier (1984), it can be followed from Fig. 7b that the lateral strain at free surface is considerably underestimated most likely caused by surface unloading during cyclic test.

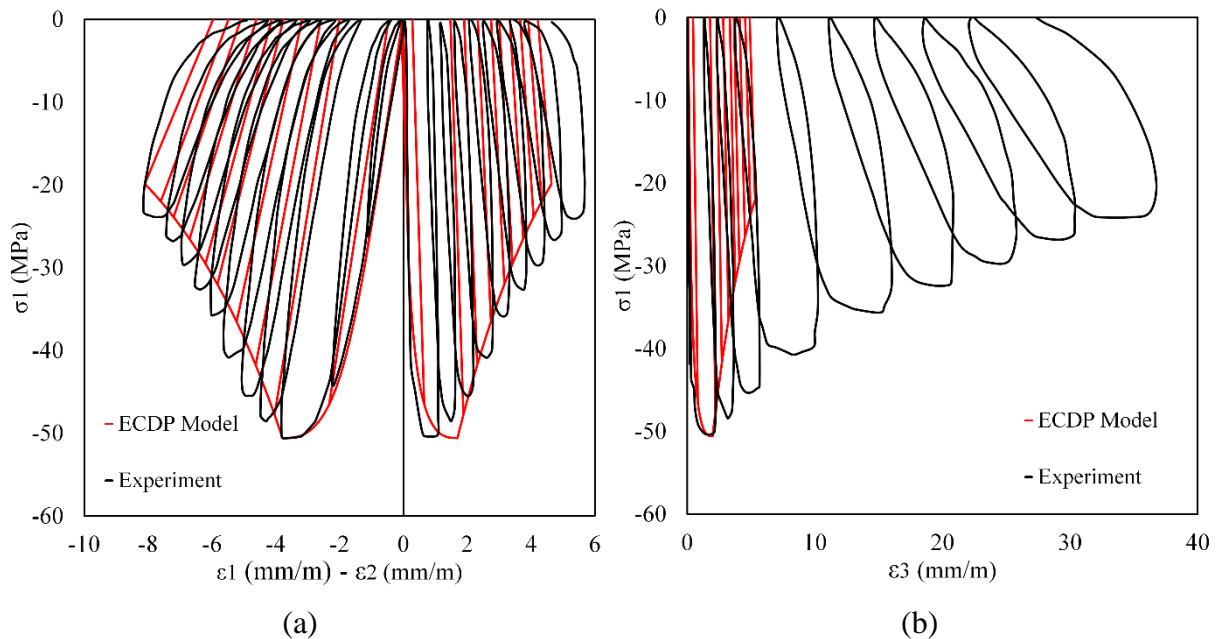


Figure 7 – Comparison of measured and computed cyclic behavior of concrete in biaxial compression test under cyclic loading conducted by Van Mier (1984).

It should be noted that, the characteristic length l_{char} is taken 100 mm for simulating both tested specimens, which is equal to the reported length of specimens.

Triaxial compression tests under cyclic loading

From structural engineering point of view, a concrete specimen is subjected to triaxial loading including gravity force, lateral confinement by stirrups and lateral loads like ground motions. Hence, this subsection evaluates the capability of ECDPM to simulate the cyclic behavior of concrete subjected to triaxial loading conditions. In this regard, two triaxial cyclic compressive tests, conducted by Imran and Pantazopoulou (1996) and Van Mier (1984), are employed.

As the first example in this subsection, the triaxial compression tests by Imran and Pantazopoulou (1996) is considered. In these tests, the impact of various confining pressures, from 0 to 43 MPa, on ultimate strength and ductility of concrete is examined. The material parameters are reported as $E = 30$ GPa, $\nu = 0.20$, $f_{cu} = 47.4$ MPa, $f_{tu} = 4.74$ MPa and G_{FI} is estimated as 0.15 N/mm. The comparison of results in Fig. 8 demonstrates that the model captures the concrete strength increasing caused by lateral confinement pressures very well.

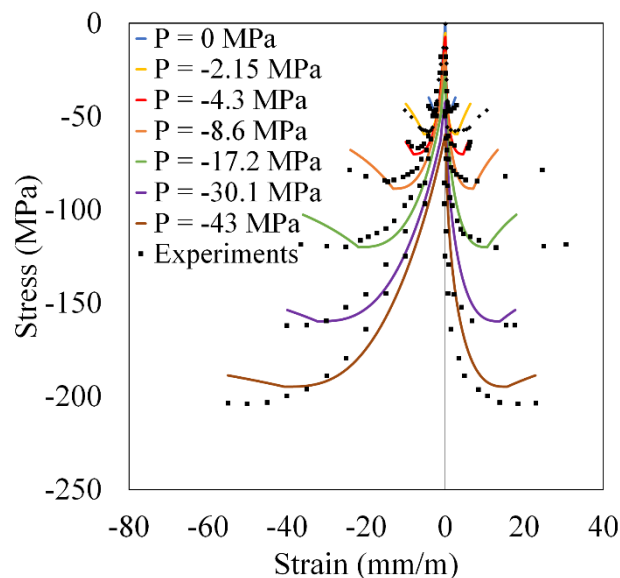


Figure 8 – Comparison of measured and computed cyclic behavior of concrete in triaxial compression tests under monotonic loading tested by Imran and Pantazopoulou (1996).

The second example focuses on investigating the capability of ECDPM to predict the cyclic behavior of plain concrete subjected to lateral confinement pressures. The mechanical properties of concrete for numerical simulation are given as $E = 27$ GPa, $\nu = 0.20$, $f_{cu} = 45.3$ MPa, $f_{tu} = 2.8$ MPa, G_{FI} is estimated as 0.15 N/mm. In this test, the lateral confining pressures were applied to the specimen as 10% of the ultimate uniaxial compressive strength of concrete (Van Mier 1984). Fig. 9 demonstrates that the concrete responses including the ultimate compressive strength and its increasing caused by lateral confinement, the softening response and also the lateral strains are predicted very well by ECDPM. The latter can be described as there were no free surface in this test, unlike two previously described tests reported by Van Mier (1984).

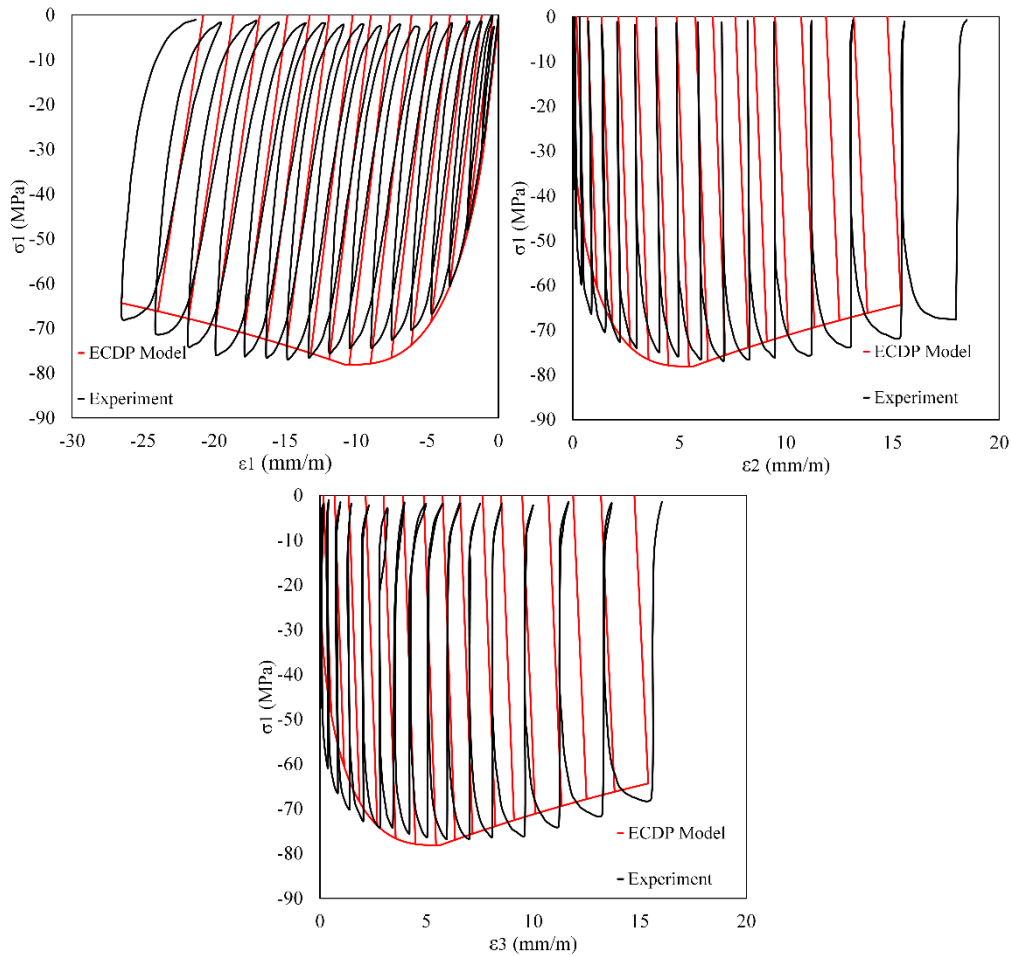


Figure 9 – Comparison of measured and computed cyclic behavior of concrete in triaxial compression test under cyclic loading conducted by Van Mier (1984).

The characteristic length l_{char} is taken 100 mm for simulating the aforementioned tested specimens, that is equal to the reported length of specimens.

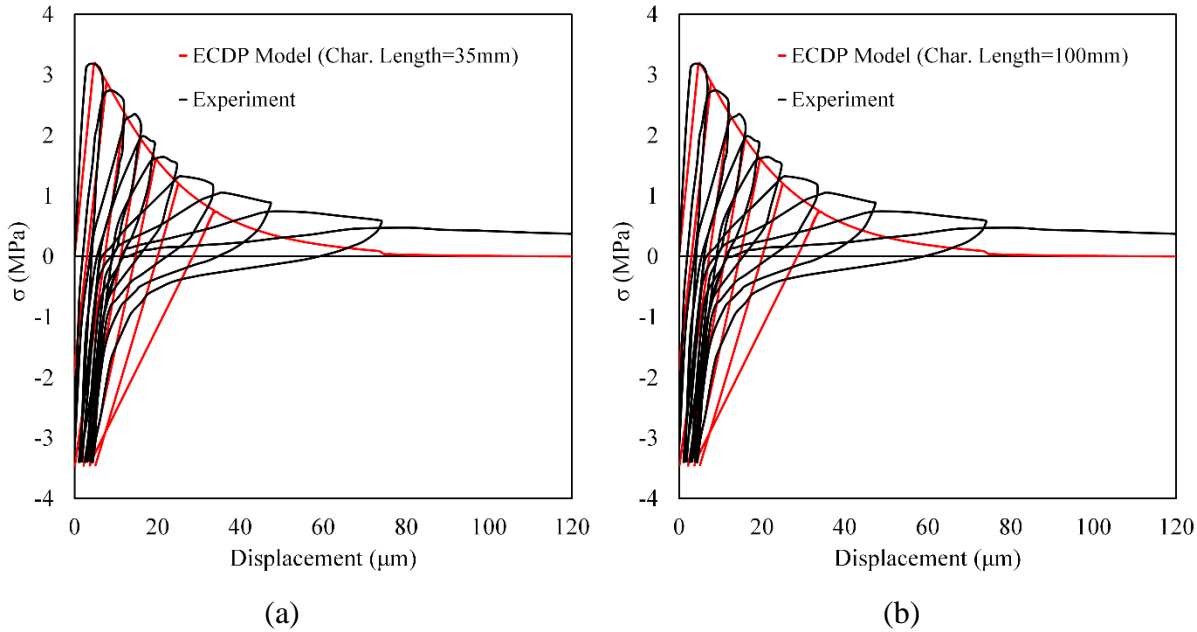
Uniaxial cyclic tension-compression test

This subsection evaluates the performance of ECDPM to simulate the cyclic response of concrete by means of a uniaxial tension-compression test conducted by Reinhardt et al. (1986). The predicted results are also compared to the ones computed by three other damage plasticity models, CDP model by Lee and Fenves (1998), CDPM1 by Grassl and Jirásek (2006) and CDPM2 by Grassl et al. (2013). In this test, the specimen attains its ultimate tensile strength, while it remains within the elastic domain in compressive regime. It is worth noting here that this is the only test in the literature, which reveals the influence of tensile damage on compressive behavior of concrete.

The capability of the CDPM1 by Grassl and Jirásek (2006) to predict the behavior of plain concrete and reinforced concrete structures subjected to monotonic loading is demonstrated in [Azadi et al. 2018]. Albeit, this model is only applicable for pushover analyses since it employs only a single damage variable for both tensile and compressive regimes. On the other hand, the CDP and CDPM2 models have demonstrated their performance to predict pure tensile or compressive behavior of concrete under cyclic or monotonic loading patterns with multiaxial loading conditions. However, as described before these models can not capture the

effects of tensile/compressive damage on compressive/ tensile response, which will be demonstrated in the following.

In this test, the material parameters are set as $E = 24 \text{ GPa}$, $\nu = 0.20$, $f_{cu} = 47.1 \text{ MPa}$, $f_{tu} = 3.2 \text{ MPa}$, $G_{FI} = 0.15 \text{ N/mm}$. The characteristic length l_{char} is taken 35 mm for simulating the tested specimen, which is equal to the reported length of specimen. Moreover, the predicted result by employing the $l_{char} = 100 \text{ mm}$ is also presented. The comparison of the predicted results by ECDPM, CDP, CDPM1 and CDPM2 with the experimental results is displayed in Fig. 10. It can be seen in Fig. 10a that the cyclic responses of concrete including the ultimate tensile strength of concrete, the softening response and the strains in compressive regime are captured very well by ECDPM with $l_{char} = 35 \text{ mm}$. On the other hand, ECDPM with $l_{char} = 100 \text{ mm}$ predicts the ultimate tensile strength of concrete and the strains in compressive regime very well, but the post-peak response is much softer than the reported response. Fig. 10c displays the predicted cyclic responses by CDP model and CDPM2 (Lee and Fenves 1998; Grassl et al. 2013). It can be followed that these models compute the ultimate tensile strength and softening response very well. Albeit, they considerably overestimate the strain in compressive regime caused by employing independent damage variables in tension and compression. The same performance can be seen for CDPM1 as shown in Fig. 10d.



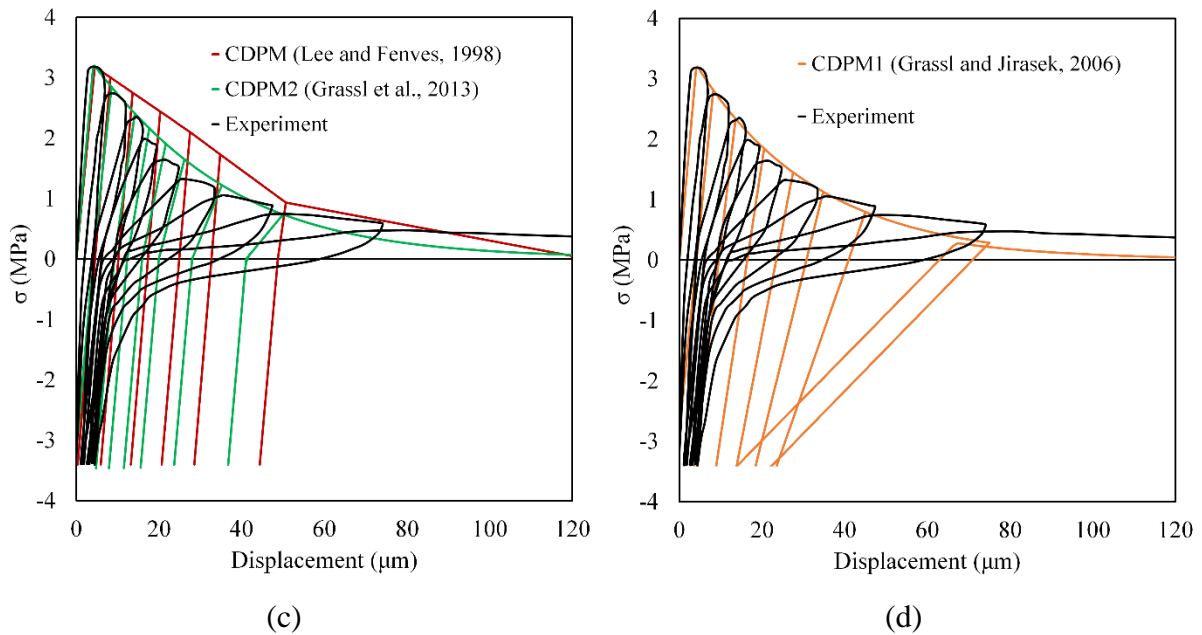


Figure 10 – Comparison of measured and computed cyclic behavior of concrete in uniaxial tension-compression, (a) ECDPM with $l_{char} = 35$ mm, (b) ECDPM with $l_{char} = 100$ mm, (c) CDP model by Lee and Fenves (1998) and CDPM2 by Grassl et al. (2013), (d) CDMP1 by Grassl and Jirásek (2006) with the experimental test conducted by Reinhardt et al. (1986).

Summary and conclusion

In this study, an Enhanced Damage Plasticity Model, called as ECDPM, is proposed for simulating the cyclic response of plain concrete under multiaxial loading conditions in tension and compression (i.e. uniaxial, biaxial and triaxial). The proposed model is developed by combination of theory of plasticity and continuum damage mechanics theory. To capture the effect of tensile or compressive damage on compressive or tensile behavior of plain concrete, two damage variables together with a strain-based weight factor are employed. In the following, several experimental tests including uniaxial cyclic and monotonic tests in tension or compression, biaxial and triaxial cyclic and monotonic compression tests are used to evaluate the performance of ECDPM. The obtained results demonstrated that ECDPM can describe the cyclic response of plain concrete under multiaxial loading conditions very well and is superior to the other previously proposed damage plasticity models. From earthquake engineering application point of view, ECDPM can accurately predict the cyclic response of concrete subjected to lateral confinement and its effects on ultimate strength and ductility capacities. It was also observed that the tensile and compressive softening response after attaining the peak strength, caused by cracking in tension and crushing in compression, was captured very well by ECDPM. Albeit, the lack of experimental data from cyclic tension-compression can be mentioned as a main problem in developing and validating more accurate models than ECDPM.

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Chapter 2: Data-driven predictive models for ultimate lateral shear strength of rectangular and circular reinforced concrete columns

In this chapter, novel data-driven models are proposed to predict the ultimate shear strength in rectangular and circular reinforced concrete columns. The models are developed using statistical methods including linear and nonlinear analyses and then validated by comparing the predicted results with experimental data, which will be discussed in the following.

Abstract

To perform seismic design of reinforced concrete (RC) structures, one of the important parts can be named as predicting the ultimate shear capacity of structural elements subjected to axial and/or lateral loads. The substantial role of columns for load distribution and providing structural stability during earthquakes, has always received many attentions in the past research activities (i.e. analytical, numerical and experimental). Albeit, many uncertainties still exist regarding the predictions by empirical and numerical models. Hence, this study aims to propose two novel numerical models for predicting the ultimate shear capacity of rectangular and circular RC columns. In this regard, two experimental databases for both aforementioned types of columns are employed in this study. Each database is randomly divided into two sets, the first set for proposing the model, called as training set and the second one for validating the proposed model, called as testing set. Finally, the obtained results were validated by means of testing sets and compared to other empirical technique and a concrete regulation. The comparison of predicted and measured results demonstrated the better accuracy of the proposed models for predicting the ultimate shear strength in both types of columns compared to the other empirical model and a concrete regulation.

Introduction

Seismic design of RC columns has been assessed in many numerical and experimental studies. However, the observed defects in seismic behavior of structural elements during past earthquakes require more studies. A proper design should provide enough strength and ductility capacity for elements to sustain the imposed internal and external loads. As a matter of fact, various geometry and mechanical properties of structural components is a challenge regarding accurate estimation of the strength or ductility capacity of structural elements. Hence, it demonstrates the necessity of a reliable model, which makes practicing engineers and researchers enable to accurately predict the behavior of RC structures subjected to different loading patterns.

Among main load-bearing structural components, this study focuses on RC columns due to their vital role in transferring axial and lateral loads and also providing structural stability. It was demonstrated that the lateral ductility and strength capacities can be employed for evaluating the response of columns during earthquakes. The relationship between lateral ductility and geometrical and mechanical properties for RC columns was investigated in some numerical and experimental research studies [Pujol 2002; Wu et al. 2009; Yavari 2013; Henkhaus 2013; Farahmand et al. 2015; Azadi and KhanMohammadi 2018]. In addition, the relevance between the lateral ductility capacity and the failure mode of RC columns was assessed in several studies [Elwood 2004; Tran and Li 2013; Azadi and Allahvirdizadeh 2018]. A number of geometrical and mechanical parameters contributes to provide the lateral strength capacity of RC columns, also called as shear strength [Xiao et al. 1999; Saatcioglu and Grira 1999; Paultre 2001; Sezen and Moehle 2004; Altin et al. 2008]. It was demonstrated by numerical and experimental studies that the transverse reinforcement and axial load ratios play a main role for providing shear strength in RC columns [Bayrak and Sheikh 1996;

Wehbe et al. 1999; Legeron and Paultre 2000; Azadi et al. 2018; Adibi et al. 2018]. The influence of the aforementioned parameters on the seismic behavior of RC columns with wide spacing of stirrups on ultimate lateral displacement and shear strength capacities was investigated in some experimental studies [Ghee et al. 1989; Mo and Wang 2000; Nakamura and Yoshimura 2002; Ranf et al. 2006; Wu et al. 2009; Yavari et al. 2013]. Moreover, the seismic performance of high strength concrete and bridge columns was also evaluated in some studies [Wong et al. 1993; Lynn et al. 1996; Priestley and Benzoni 1996; Xiao and Martirosyan 1998; Saatcioglu and Baingo 1999; Pandey and Mutsuyoshi 2005; Yi et al. 2018]. Recently, Aval et al. (2017) proposed a data-driven model for predicting shear strength in short RC columns by means of regression analysis.

The aim of this study is to identify the ultimate shear strength in rectangular and circular RC columns. To this purpose, two extensive experimental databases for each types of columns are used, which are divided into two sets; training and testing sets. The most effective geometrical and mechanical parameters are selected and then linear and nonlinear analyses were performed to derive the numerical models for predicting the shear strength in RC columns using the training dataset. The capability of the proposed models was evaluated by comparing the predicted results with testing datasets. Moreover, the predicted results were also compared to the ones computed by an empirical model proposed by Sezen and Moehle (2004) for rectangular columns and ACI 318-14 concrete provisions for both types of columns. It was observed that the proposed linear equations have better performance compared to the nonlinear equations and also other models for predicting the ultimate shear strength in both rectangular and circular columns. Hence, these equations are proposed as the final equations in this study.

Previous models for calculating the shear strength capacity in RC columns

There are several models in the literature for predicting the shear strength in RC columns, which are developed based on empirical, numerical and experimental studies. These models are proposed using the geometrical details of columns' sections and mechanical properties of concrete and reinforcements. Some of the most practical models in engineering calculation for the shear strength in rectangular RC columns are presented below.

Priestley et al. (1994) proposed a model to calculate the shear in columns subjected to cyclic loading. In addition the contribution of concrete, the model can also describe the influence of transverse reinforcement and the arch mechanism caused by compressive axial load on ultimate shear strength, which takes the form

$$V_n = V_c + V_s + V_p$$

$$V_c = k\sqrt{f'_c}0.8A_g \text{ (MPa)}, V_s = \frac{A_{st}f_{yt}D'}{s} \cot 30^\circ, V_p = \frac{h - c}{2a} P, \quad (5)$$

where, V_c is the shear strength provided by concrete, V_s is the shear strength provided by transverse reinforcement, V_p the shear strength provided by the arch mechanism, the parameter k is a function of displacement ductility, f'_c is the compressive strength of concrete, A_g is the area of column section, A_{st} and f_{yt} are the area and yield strength of stirrups, D' is distance measured parallel to the applied shear between centres of the perimeter hoop, s is the spacing of transverse reinforcement, h is the column section height, c is the neutral axis depth, a is the distance from maximum moment section to point of inflection, P is the compressive axial load.

FEMA 273 Seismic Rehabilitation Guidelines (1997) recommended another model for estimating the shear in RC columns, which reads

$$V_n = V_c + V_s$$

$$V_c = \alpha \lambda \left(k + \frac{P}{\beta A_g} \right) \sqrt{f'_c} b d, V_s = 0.5 \left(\frac{A_{st}}{s} \right) f_{yt} d, \quad (6)$$

where, α and β denoting the constant coefficients, λ is the modification factor related to unit weight of concrete, b and d are the width and effective depth of column section, respectively. One of the most popular and practical models in the literature, which is widely used in this study, was proposed by Sezen and Moehle (2004) and it can be described as follows

$$V_n = V_c + V_s$$

$$V_c = k \left(\frac{\alpha \sqrt{f'_c}}{a/d} \sqrt{1 + \frac{P}{\beta \sqrt{f'_c} A_g}} 0.8 A_g \right), V_s = k \frac{A_{st} f_{yt} d}{s}, \quad (7)$$

wherein, the first parameter k is a factor for concrete contribution that is one for the displacement ductility less than 2 and is 0.7 for the displacement ductility exceeding 6, α and β are the constant coefficients, a/d is the column aspect ratio and the second parameter k is the factor for stirrups contribution. Another practical model is proposed by ACI 318-14 concrete provisions, which takes the form

$$V_n = V_c + V_s$$

$$V_c = \alpha \left(1 + \frac{P}{\beta A_g} \right) \sqrt{f'_c} b d, V_s = k \frac{A_{st} f_{yt} d}{s}, \quad (8)$$

where, α and β denoting the constant coefficients.

Next, the available models in the literature for predicting shear strength of circular RC columns are presented in the following. Kowalsky and Priestley (2000) proposed a model to calculate the shear strength in circular RC columns. In this model the shear strength is provided by concrete, transverse reinforcement and the arch mechanics caused by axial load, which reads

$$V_n = V_c + V_s + V_p$$

$$V_c = \alpha \beta \gamma \sqrt{f'_c} 0.84 A_g, V_s = \frac{\pi A_{st} f_{yt}}{4 s} (D - c - cc), V_p = \frac{h - c}{2L} P, \quad (9)$$

In Eq. 9, α , β and γ depend on columns aspect ratio, the longitudinal reinforcement ratio and ductility, respectively, cc is the concrete cover of column section and L is the height of column. Next, the proposed model by ACI 318-14 concrete provisions predicts the shear capacity by contribution of concrete and stirrups, which can be described as follows

$$V_n = V_c + V_s$$

$$V_c = a_1 \left(1 + \frac{P}{a_2 A_g} \right) \sqrt{f'_c} b_v d_v, V_s = \frac{A_{st} f_{yt} d_v}{s}, \quad (10)$$

where, a_1 and a_2 are constant coefficients, b_v and d_v are web width and effective shear depth of column section. The last model in section is recommended by AASHTO LRFD Bridge Design Specifications (2014), which takes the form

$$V_n = V_c + V_s$$

$$V_c = 0.0316\beta\sqrt{f'_c}b_vd_v \text{ (ksi)}, V_s = \frac{A_{st}f_{yt}d_v}{s} \cot \theta \text{ (ksi)}, \quad (11)$$

wherein, β in this model denotes a factor indicating ability of diagonally cracked concrete to transmit tension and shear and θ is the angle of inclination of diagonal compressive stresses.

Proposed models for shear strength in rectangular and circular columns

This section is devoted to present the proposed models for predicting the ultimate shear strength in rectangular and circular RC columns.

The proposed models for rectangular columns and their validation with experimental results

In this subsection, the proposed models for calculating shear strength in rectangular RC columns are presented and compared to an empirical model (Sezen and Moehle 2004) and a concrete regulation (ACI 318-14). These models are developed by contributing geometrical details of columns and mechanical properties of concrete and steel materials. To this aim, an extensive database including tested rectangular RC columns is used. The most important parameters with their range (minimum and maximum), mean value and standard deviation are presented in Table 1.

Table 1- Statistical data of the employed parameters for tested rectangular RC columns

Parameter (Unit)	Range (Minimum-Maximum)	Mean	Standard Deviation
ρ_t (%)	(0.1-3.2)	0.7	0.5
ρ_l (%)	(0.7-6.9)	2.4	1.0
a/d (-)	(1.2-9.0)	3.8	1.6
f'_c (MPa)	(13-118)	46	28
f_{yt} (MPa)	(249-1424)	469	204
f_{yl} (MPa)	(318-587)	428	62
$P/(A_g f'_c)$ (-)	(0-0.9)	0.2	0.2
V_{test} (kN)	(37-1339)	218	172

where, ρ_t and ρ_l denote the transverse and longitudinal reinforcement ratios, respectively, f_{yl} is the yield strength of longitudinal reinforcement, $P/A_g f'_c$ is the initial axial load ratio and V_{test} is the reported shear strength during tests. It should be mentioned that there are some material strength limitations by ACI 318-14, which can be stated as the compressive strength of concrete is well below 70 MPa and the yield stress of transverse and longitudinal reinforcements shall not exceed 420 MPa. Moreover, it is recommended by ASCE-ACI Committee 426 report (1974) that $2 \leq a/d \leq 4$.

In order to develop accurate and reliable models for predicting shear strength in rectangular and circular columns, three parametric regression analyses including linear and nonlinear analyses were performed. The nonlinear analyses were consisted of quadratic equations with and without mixed terms. The analyses yielded polynomial equations in terms of geometrical and mechanical terms, which can be generally given as

$$V_n = \alpha + \sum_{i=1}^n (\beta_i X_i + \gamma_i X_i^2) + \sum_{i=1}^n \sum_{j=1}^n \lambda_{ij} X_i X_j \quad (12)$$

$$X_1 = f'_c b d \left(\frac{P}{A_g f'_c} \right), \quad X_2 = f'_c b d \left(\frac{d}{a} \right), \quad X_3 = A_{st} f_{yt}, \quad X_4 = A_{sl} f_{yl}$$

where, α , β_i , γ_i and λ_{ij} are the regression coefficients reported by regression analyses, X_i and X_j are the model parameters and A_{sl} denoting the area of the longitudinal reinforcement. It should be mentioned here that to perform linear regression analyses $\gamma_i = \lambda_{ij} = 0$ and to conduct quadratic regression analyses without mixed terms $\lambda_{ij} = 0$. Due to positive influence of the model parameters in Eq. 12 on the shear strength capacity, the model parameters with negative regression coefficients were ignored and the linear regression analysis was performed again with remained material parameters, which was observed for the term of transverse reinforcement. Furthermore, the model parameters with significant factor (Sig. factor) greater than 0.05 were omitted for the next regression analyses since they are not good predictors. Hence, these model parameters are absent in the final equation for rectangular columns, which will be described in the following.

In order to performing regression analyses and consequently determining the regression coefficients, the employed experimental database was randomly categorized into two sets as training group (70% of the database) and testing group (30% of the database). Next, the linear and nonlinear analyses were performed using SPSS package [SPSS 2017]. The reported results for linear and nonlinear analyses with coefficient of determination (R^2) and test set error for both first and second testing sets are presented in Table 2 and compared to the predicted results by Sezen and Moehle (2004) and ACI 318-14. It is worth noting here that the second testing set was collected using the tested rectangular columns, which are not available in the main employed database. It should be noted that R^2 varies between 0 (no correlation) to 1 (best correlation).

Table 2 - Statistical data of the parameters for shear strength in rectangular columns

Method	R^2	First test set error (%)	Second test set error (%)
Linear model	0.858	23	16
Quadratic model (without mixed terms)	0.882	28	18
Full quadratic model	0.900	27	31
Sezen and Moehle (2004)	0.500	76	60
ACI 318-14	0.409	117	73

It can be observed in Table 2 that the reported R^2 is greater than 0.85 for the linear and two nonlinear models, which demonstrates the high accuracy of the proposed models compared to the other models by Sezen and Moehle and ACI 318-14. Furthermore, the computed test set errors for the linear equation are 23% and 16% for the first and second testing sets, respectively, which are less than the proposed nonlinear models and other techniques. Thereby, the linear model is selected as the final model for predicting the shear strength in rectangular RC columns and can be given as follows:

$$V_{shear} = f'_c b d \left(0.035 \frac{P}{A_g f'_c} + 0.135 \frac{d}{a} \right) + 0.054 A_{sl} f_{yl} \quad \text{kN or kips.} \quad (13)$$

In order to calculate the shear in kN or kips the material parameters should be inserted into Eq. 13 as kN-mm or kips-inch, respectively.

The superiority of the proposed linear equation over the nonlinear models and other approaches is also demonstrated in Fig. 11. In this regard, the predicted results by proposed

models and other techniques were compared to the testing sets. It can be followed from Fig. 11 that there is large scatter of the ratio of predicted to the measured shear for the models by Sezen and Moehle (2004) and ACI 318-14. Meanwhile, the capability of the proposed models to predict shear strength, compared to the experimental results, is demonstrated. Finally, the linear model is proposed due to less test errors compared to the other quadratic models, as presented in Table 2.

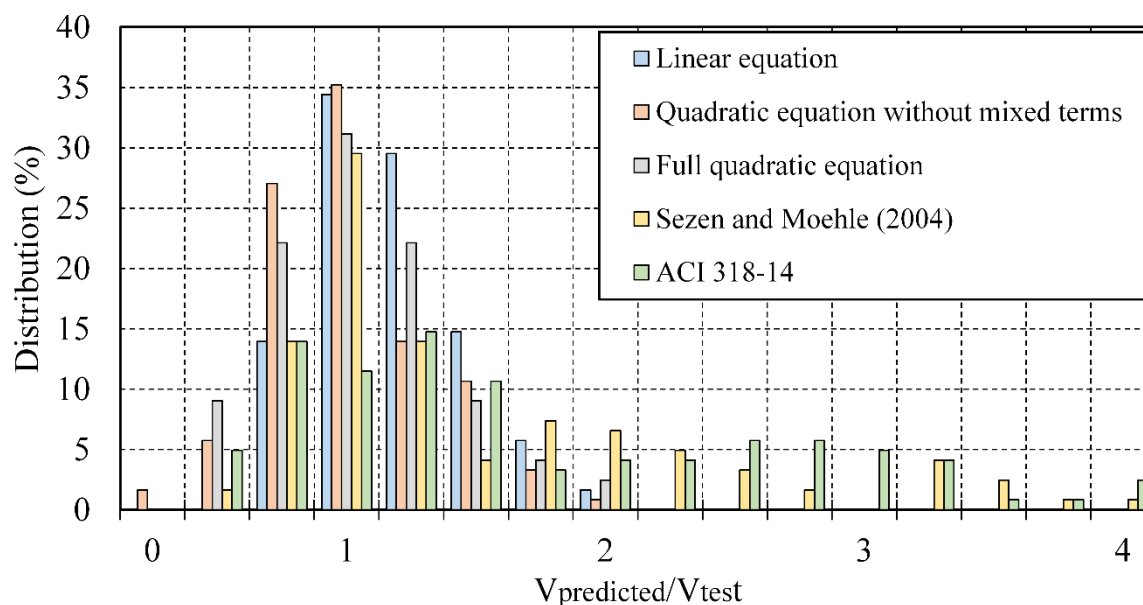


Figure 11 – Comparison of distribution versus the ratio of predicted to measured shear strength in rectangular RC columns for proposed linear and nonlinear methods and the models by Sezen and Moehle (2004) and ACI 318-14 by means of two testing sets.

The proposed model for circular columns and their validation with experimental results

This subsection is devoted to present the proposed models for calculating shear strength in circular RC columns and compare them to experimental data and the model by ACI 318-14. The same as the previous section for rectangular columns, the new models for circular columns are developed in terms of geometrical details of columns and mechanical characteristics of concrete and reinforcing steel. To this purpose, a large database consisting of the tested circular columns is employed. The most important parameters with their statistical data including their range (minimum and maximum), mean value and standard deviation are presented in Table 3.

Table 3- Statistical data of the employed parameters for tested circular columns

Parameter (Unit)	Range (Minimum-Maximum)	Mean	Standard Deviation
ρ_t (%)	(0.04-1.60)	0.4	0.31
ρ_l (%)	(0.5-5.6)	2.5	1.03
a/d (-)	(1.3-12.5)	4.2	2.42
f'_c (MPa)	(19-90)	37	14
f_{yt} (MPa)	(200-1000)	416	139
f_{yl} (MPa)	(240-565)	416	63
$P/(A_g f'_c)$ (-)	(0-0.74)	0.14	0.14
V_{test} (kN)	(19-3282)	281	312

As stated in the previous section, ACI 318-14 concrete provisions require some material strength limitations; the compressive strength of concrete $f'_c \leq 70$ MPa and the yield stress of

transverse and longitudinal reinforcements $f_{yt} \leq 420$ MPa and $f_{yl} \leq 420$ MPa, respectively. Furthermore, the column aspect ratio is also limited as $2 \leq a/d \leq 4$ by ASCE-ACI Committee 426 report (1974).

To propose the linear and nonlinear models the same procedure was followed as the same as the previous section for rectangular columns. Three parametric linear and nonlinear regression analyses were performed. The polynomial equations can be described as follows

$$V_n = \alpha + \sum_{i=1}^n (\beta_i Y_i + \gamma_i X Y_i^2) + \sum_{i=1}^n \sum_{j=1}^n \lambda_{ij} Y_i Y_j \quad (14)$$

$$X_1 = f'_c b d \left(\frac{P}{A_g f'_c} \right), \quad X_2 = f'_c b d \left(\frac{D}{a} \right), \quad X_3 = A_{st} f_{yt}, \quad X_4 = A_{sl} f_{yl}$$

where, Y_i and Y_j are the model parameters for circular columns and D denoting the diameter of the column section. It was observed in first linear analyses that the reported significant factor is greater than 0.05. Hence, this parameter was omitted for the next analyses and also from the final reported equation.

Next, the regression analyses were performed using training set including 70% of the database in SPSS package (SPSS 2017). The statistical data reported by regression analyses including the coefficient of determination (R^2) and test set error for the first and second testing sets are presented and compared to the ones by ACI 318-14 are presented in Table 4.

Table 4 - Statistical data of the parameters for shear strength in circular columns

Method	R^2	First test set error (%)	Second test set error (%)
Linear model	0.701	22	23
Quadratic model (without mixed terms)	0.716	28	42
Full quadratic model	0.775	24	243
ACI 318-14	0.272	111	48

It can be seen in Table 4 that the reported R^2 is greater than 0.7 for linear and two nonlinear models, which shows the high accuracy of the proposed models compared to the model by ACI 318-14. The capability of the proposed models and ACI 318-14 model was examined by means of two testing sets. Note that the first testing set was randomly selected from the main database including 30% of the tested specimens and the second testing set was collected by employing other test results out of the main database. Comparison of the computed test set errors reveals that the linear model is more accurate to predict the shear strength of circular RC columns compared to the other models. Hence, the linear model is chosen as the final model, which can be described as

$$V_n = f'_c A_g \left(0.041 \frac{P}{A_g f'_c} + 0.106 \frac{D}{a} \right) + 0.027 A_{sl} f_{yl} \quad \text{kN or kips.} \quad (15)$$

To compute the shear strength in kN or kips the material parameters should be inserted into Eq. 14 as kN-mm or kips-inch, respectively.

For better understanding the superiority of the proposed linear model over two nonlinear and ACI 318-14 models, the distribution of the ratio of predicted shear to measured shear is illustrated in Fig. 12. It can be seen in Fig. 12 that there is a large scatter in the predicted results by the full quadratic and ACI 318-14 models. While, the linear equation and quadratic

equation without mixed terms predict the measured shear strength very well. Nevertheless, the linear model is the best choice due to less test set errors as presented in Table 4.

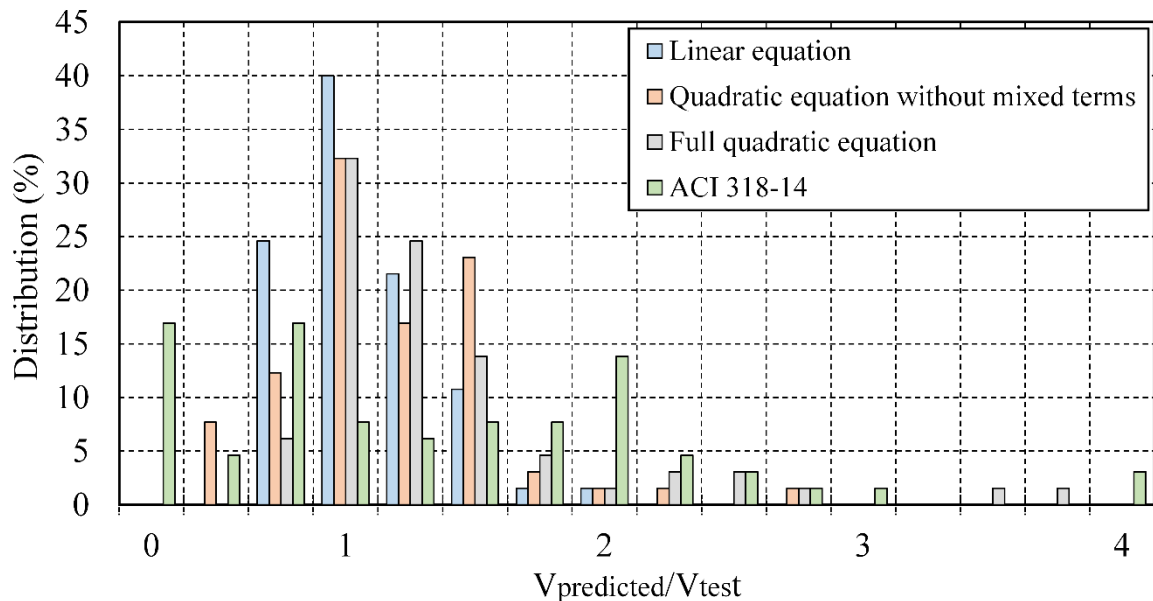


Figure 12 – Comparison of distribution versus the ratio of predicted to measured shear strength in circular RC columns for proposed linear and nonlinear methods and the model by ACI 318-14 using two testing sets.

Summary and conclusion

This study aims to propose two novel data-driven models to predict the shear strength in rectangular and circular reinforced concrete (RC) columns. To this purpose, two extensive databases are employed and randomly divided into two sets; training and testing sets. Several parametric linear and nonlinear regression analyses were conducted on the training sets to propose reliable models. In the following, the obtained results by the proposed models were validated using the testing sets (two testing sets) and then compared to other conventional model and a concrete regulation. The comparison of results demonstrated the capability of the proposed models to calculate the shear strength in RC columns compared to other techniques. Finally, the linear models were selected for both rectangular and circular columns due to less test set errors.

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